Løsningsforslag til prøveeksamen

Problem 1 Since the total claim amounts $S_i(t), i = 1, 2$ are independent compound Poisson random variables (for $t = 1$), we know that the aggregated total claim amounts

$$\widetilde{S}(t) = S_1(t) + S_2(t)$$

have compound Poisson distributed with

$$\widetilde{S}(t) \stackrel{d}{=} \sum_{j=1}^{N_\lambda} Y_j,$$

with clain numbers $N_\lambda \sim Pois(\lambda), \lambda = \lambda_1 + \lambda_2 = \frac{17}{20}$ and $i.i.d$ fire loss claim sizes Y_j specified by the "mixture distribution", that is

$$P(Y_1 = x) = p_1 P(X_1^{(1)} = x) + p_2 P(X_1^{(2)} = x),$$

where

$$p_i = \frac{\lambda_i}{\lambda_1 + \lambda_2}, \; i = 1, 2.$$

See T. Mikosch.

Therefore we get

$$P(Y_1 = x) = \begin{cases} 0 & , x \neq 1, 2, 3 \\ 5/68 & x = 1 \\ 47/68 & x = 2 \\ 4/17 & x = 3 \end{cases}$$

In this case the Panjer recursion is given by

$$p_n = \sum_{i=1}^{n} \frac{17i}{20n} \cdot P(X_1 = i) \cdot p_{n-i}, \; n \geq 1,$$

with initial value

$$p_0 = P(N_\lambda = 0) = e^{-\lambda} \approx 0.427415.$$

This implies

$$\begin{aligned} P(\widetilde{S}(1) &= 1) \approx 0.0267, P(\widetilde{S}(1) = 2) \approx 0.2519, P(\widetilde{S}(1) = 3) \approx 0.1012 \\ (, ..., P(\widetilde{S}(1) &= 10) \approx 0.00196), \end{aligned}$$

i.e.

$$\begin{aligned} P(\widetilde{S}(1) &\leq 1) \approx 0.4541, P(\widetilde{S}(1) \leq 2) \approx 0.7061, P(\widetilde{S}(1) \leq 3) \approx 0.8073 \\ (,...,P(\widetilde{S}(1) &\leq 10) \approx 0.9984) \end{aligned}$$

Problem 2

(i) Because of the equivalence principle Π_0 satisfies the equation

$$\sum_{k=0}^{3} c_{k+1} v^{k+1} \cdot_k p_x q_{x+k} + E v^4 \cdot_4 p_x - \sum_{k=0}^{3} \Pi_k v^k \cdot_k p_x = 0,$$

where $c_{k+1} = (1 + 2k), k = 0, 1, 2, 3$. Solving the latter equation gives

$$\Pi_0 = 0.699115.$$

(ii) Apply the discrete version of Thiele's differential equation, that is

$${}_kV + \Pi_k = v \left[c_{k+1} \cdot q_{x+k} + {}_{k+1}V \cdot p_{x+k}\right]$$

to obtain ${}_1V = 0.714958,{}_2 V = 1.45596,{}_3 V = 2.21893,{}_4 V = 3 = E$.

Problem 3 In the Crámer-Lundberg model the claim number process $N(t)$ is independent of the $i.i.d$ claim sizes $(X_j)_{j\geq 1}$. So

$$E\left[R_{ExL}(t)\right] = E\left[N(t)\right] E\left[(X_1 - D)_+\right]$$

for $t = 365$ (days) and the deductible $D = 3$ (mio DKK). Since $\lambda = 0.3$, we get

$$E\left[N(t)\right] = 0.3 \cdot 365 = 109.5.$$

Further we have

$$\begin{aligned} E\left[(X_1 - D)_+\right] &= \sum_{n\geq 0} E\left[\max(n-3,0) \cdot 1_{\{X_1=n\}}\right] \\ &= \sum_{n=4}^{10} (n-3) P(X_1 = n) = \frac{7}{100}. \end{aligned}$$

So

$$E\left[R_{ExL}(t)\right] = 109.5 \cdot \frac{7}{100} = 7.665$$

in mio DKK.

Problem 4 In the Cramér-Lundberg model the inter-arrival times $W_i, i \geq 1$ are $i.i.d$ with $W_1 \sim Exp(\lambda)$. The likelihood function w.r.t. $Exp(\lambda)$ is given by

$$L(\lambda) = \prod_{i=1}^{n} f(W_i),$$

where

$$f(x) = \lambda e^{-\lambda x},\ x \geq 0$$

is the density of the exponential distribution.

Then maximization of the likelihood function yields the maximum-likelihood estimator of the jump intensity $\widehat{\lambda}$ of $N(t)$, i.e.

$$\widehat{\lambda} = \frac{n}{\sum_{i=1}^{n} W_i}.$$

Hence we get for $n = 8$ og $T_8 = \sum_{i=1}^{8} W_i = 30$ (days)

$$\widehat{\lambda} = \frac{8}{30},$$