26$^{\text{th}}$ September, 2019

STK3505/STK4505

Mandatory assignment 1 of 1

Submission deadline

Thursday 10$^{\text{th}}$ 10 2019, 14:30 in Canvas (canvas.uio.no).

Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

The assignment consists of two parts. To pass, you must answer both. Students taking the STK4505 course have write the solution in LaTeX

Problem 1. Suppose a non-life insurance company has responsibility for a portfolio of $J = 1000$ policies. Assume that the number of claims $\mathcal{N}$ is Poisson distributed with intensity $\mu = 0.02$, and that the claim sizes Z_i have Inverse Gamma distribution with $\mathrm{E}(Z_i) = 10$ and $\mathrm{sd}(Z_i) = 2.0, 3.0, 4.0$. The density function of the Inverse Gamma distribution is

$$f(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} \exp(-\frac{\beta}{x}),$$

where $\alpha > 0$ is the shape parameter and $\beta > 0$ is the scale parameter. Its mean is $\frac{\beta}{\alpha-1}$ for $\alpha > 1$ and its variance is $\frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$ for $\alpha > 2$. Hint: If $X \sim \Gamma(\alpha, \beta)$, (Gamma distribution with rate parameter β)then$\frac{1}{X} \sim \mathrm{Inv}-\mathrm{Gamma}(\alpha, \beta)$. In R you may use "invgamma" package.

a) Find the parameters α and β of the Inverse Gamma distribution for each of the values of $\mathrm{sd}(Z_i)$ and plot the probability density function of each of them.

b) Compute the 95% and 99% reserve for this portfolio for each of the sets of parameters.

c) Assume that the compensation has a deductible $a = 7.0$ and a maximum insured sum $b = 12.0$, and repeat b). Compare with the results from b).

Problem 2. Consider a put option with time to maturity $T = 1$ and the underlying return R following the log-normal model with volatility $\sigma\sqrt{T}$, and assume that the risk-free return is $r = 0.04$.

a) Assume that $r_g = 0.06$ and σ varies between $0.25, 0.3, 0.35$. Compute the value of this option using the Black-Scholes formula and Monte Carlo method. Describe how you proceed and comment on the results.

b) Assume now that $\sigma = 0.25$ and let r_g vary between $0.03, 0.06, 0.09$. Compute the value of this option using both methods and comment on the results.

c) Consider now a cliquet option with the same conditions as in b), and with $r_c = 0.1, 0.2, 0.3, 0.4$. Compute the value of the option for different values of r_g and r_c. Describe how you proceed and compare to the results from b).