Problem 1

a)Parmeters for $R_{0:K}$ are $K\xi$ and $\sqrt{K}\sigma$.

b) Insert $T = K$ in the Black-Scholes formula.

c) The fee for the option is precisely $\pi(1)v_0'$ and the initial capital is therefore lowered to $v_0/\{1+\pi(1)\}$ which earns the best of $R_{0:K}$ and r_g.

d) A pseudo R-program:
$\sigma_x = \sigma/\sqrt{1-a^2}$
X=rep(r*exp($-\sigma_x^2$)),m)
R=rep(1,m)
for (k in 1:K){
$X = X*a + \sigma*$rnorm(m)
$R = R*(1 + r*$exp$(-\sigma_x^2/2 + X))$}

e) Take the mean, sd and the 5% smallest/largest of the simulations.

f) The floating rate yields a more stable performance at somewhat lower average, but can't cope with the very high returns one sometimes obtains in the stock market. Using put options stabilizes, but apart from some occasional high returns seem to perform inferior to the floating rate.

Problem 2

a) $a_{l_0} = \sum_{k=l_r-l_0}^{\infty} {}_kp_{l_0} d^k$

b) $s = V/a_{l_0}$.

c) $E(s) = v_0 e^{K\xi + K\sigma^2/2}/a_{l_0}$ and $\mathrm{sd}(s) = E(s)\sqrt{e^{K\sigma^2} - 1}$

d) Means are 0.286, 0.239, 0.198, 0.237 and standard deviations 0.452, 0.337, 0.214, 0.026 in the order of the table in Problem 1.

e) Present values 0 of premia and pension. The mathematical expression is

$$\zeta = s\frac{\sum_{k=l_r-l_0}^{\infty} {}_kp_{l_0} d^k}{\sum_{k=0}^{l_r-l_0-1} {}_kp_{l_0} d^k}$$

f) Lowest values of ζ with lowest discounts. A mathematical argument is to write

$$\zeta = s\frac{\sum_{k=l_r-l_0}^{\infty} {}_kp_{l_0} d^{k-l_r+l_0+1}}{\sum_{k=0}^{l_r-l_0-1} {}_kp_{l_0} d^{k-l_r+l_0+1}}$$

and the numerator is an increasing and the denominator a decreasing function of d.

Problem 3

a) If U is a vector of uniforms, then $Z = \beta*((\log(U))^{-1/alpha})$ is a vector generated by the inversion sampler.

b,c) A pseudo R-program:
N=rpois(m,λ)
X=rep$(m,0)$
for (i in 1:m){
    U=runif(m)
    $Z=\beta$*$((-log(U))^{-1/\alpha})$
    Zce=Z-pmax$(Z-a,0)$*θ
    $X[i]$=sum(Zce)}

d) 83.0 and 112.7 without and 67.6 and 83.1 with reinsurance at level 0.05 and 0.01.

e) The pure premium is 5.8 and the real one 11.6.

f) From the formulae in the beginning is sd proportional to $\sqrt{J}$ and the mean proportional to J and so the ratio tends to 0 as $J \to \infty$.

g) $E(\mathcal{X}|\mu) = J\mu\xi_z$ and $\text{var}(\mathcal{X}|\mu) = J\mu(\xi_z^2 + \sigma_z^2)$ and the rule of double expectation yields $E(\mathcal{X}|\mu) = JE(\mu)\xi_z$ whereas

$$\text{var}(\mathcal{X}) = \text{var}(J\mu\xi_z) + E\{J\mu(\xi_z^2 + \sigma_z^2)\} = J^2\xi_z^2\text{var}(\mu) + JE(\mu)(\xi_z^2 + \sigma_z^2)\},$$

and the standard deviation has a term proportional to J as the leading one.