UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in: STK3505/4505 — Problems and methods in Actuarial science

Day of examination: 15th December 2020

Examination hours: 15:00 – 19:00

This problem set consists of 4 pages.

Appendices: None

Permitted aids: All examination aids are allowed

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 is on general insurance, Problem 2 on life insurance and Problem 3 on financial risk. Points for each sub-problem are given next to the number of the sub-problem. Write program sketches either in pseudo code or as R commands.

Problem 1 General insurance

Let $\mathcal{X} = Z_1 + \cdots + Z_{\mathcal{N}}$ be the pay-out in a general insurance portfolio under standard assumption, where $\mathcal{N} \sim Poisson(\lambda)$ is independent of the individual losses Z_i, which are exponentially identically distributed and independent.

$$E(Z) = \mu,$$
$$Var(X) = \mu^2,$$
$$median(X) = \mu \ln(2),$$
$$f(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$$

Assume that for a given set of historical losses $z_1, \ldots, z_n$, the sample median is 3.8216.

1a 4pt

Find the moment estimate of μ and justify your answer.

1b 4pt

Find the expression for the maximum likelihood estimator of μ.

(Continued on page 2.)

1c 4pt

Derive a sampler for the distribution using the inversion method.

1d 6pt

Sketch a program for generating m samples of $\mathcal{X}$. Explain how to determine $E(\mathcal{X})$ and $Var(\mathcal{X})$, as well as the reserve from these simulations. Define what the reserve is.

The table below shows the expectation, standard deviation and a few percentiles of the distribution of $\mathcal{X}$ when $\lambda = 20$. The number of simulations was 100000:

$E(\mathcal{X})$	$sd(\mathcal{X})$	1%	5%	25%	50%	75%	95%	99%
110.17	34.94	41.41	57.88	85.28	107.49	131.96	172.05	203.62

1e 6pt

Derive expressions for $E(\mathcal{X})$ and $sd(\mathcal{X})$ using the rules of double expectation and variance. Compare the exact values of $E(\mathcal{X})$ and $sd(\mathcal{X})$ with the ones from the table. Are there any signs of errors in the simulation program?

1f 1pt

What are the 95% and 99% reserves of the portfolio?

1g 2pt

Find the premium for loadings $\gamma = 0.3$.

Suppose the portfolio is reinsured through a contract where the reinsurer responsibility is

$$Z^{re} = aZ$$

where $0 < a < 1$ is fixed by the contract.

1h 2pt

Show that $\mathcal{X}^{re} = a\mathcal{X}$.

1i 2pt

Find the 95% and 99% and pure premium of the cedent for $a = 0.4$.

(Continued on page 3.)

Problem 2 Life insurance

Age: x - $x+1$	Men	Women	General population
$59-60$	0.010619	0.006429	0.008456
$60-61$	0.011470	0.006880	0.009093
$61-62$	0.012361	0.007371	0.009768
$62-63$	0.013260	0.007903	0.010467
$63-64$	0.014140	0.008481	0.011181
$64-65$	0.015019	0.009111	0.011922
$65-66$	0.015942	0.009793	0.012710

Table 1: Probability of dying between ages x and x + 1

Suppose a pension for person of age l_0 starts to run at age l_r, lasting until the individual dies. Until the retirement the individual contributed amount π every year.

2a 3pt

Calculate the probability of surviving from age 60 to 65 for the general population.

2b 5pt

For $l_0 = 60$, $l_r = 65$, $\pi = 1000$ and interest rate $r = 5\%$ compute the value of the insurance policy at the end of the contributing stage. Use life table for the general population. How this value will compare if you consider men and women separately?

2c 4pt

Consider now an insurance that releases upon death. Design a program that simulates the number of paid out insurances in a two year time frame. Start with 1000 individuals of age 60, where 50% is men and 50% is women.

Problem 3 Financial risk

Consider a call option written on R with pay-off $X_C = \max(R - r_g, 0)v_0$, where $R = e^{\xi_q T + \sigma\sqrt{T}\epsilon} - 1$, $\epsilon \sim N(0,1)$, r_g is the minimum return, r is a risk-free interest rate, T is the time period and ξ_q, σ are parameters.

(Continued on page 4.)

3a 2pt

What theoretical premium does a call option have?

3b 10pt

Derive the Black-Scholes formula for the call option premium.

Consider a portfolio of two assets R_1, R_2 where $R_i = e^{\xi_{q_i} T + \sigma_i \sqrt{T} \epsilon_i} - 1$ and ϵ_1, ϵ_2 are correlated with correlation parameter ρ.

3c 7pt

Sketch an algorithm for simulating the premium for a call option with pay-off $X = \max(\mathcal{R} - r_g, 0)v_0$ written on the portfolio $\mathcal{R} = w_1 R_1 + w_2 R_2$ if the weights are $1/3$ and $2/3$. What effect of the correlation do you expect?

3d 6pt

Assume now that $\rho = 0$. Sketch an algorithm for simulating the premium for an option written on the difference between R_1 and R_2, that is with pay-off $X = \max(R_1 - R_2, 0)v_0$.

END