5th October, 2021

STK3505/STK4505

Mandatory assignment 1 of 1

Submission deadline

Thursday 21st 10 2021, 10:00 in Canvas (canvas.uio.no).

Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

The assignment consists of two parts. To pass, you must answer both. Students taking the STK4505 course have to write the solution in $\LaTeX$. Include all relevant code.

Problem 1. Suppose a non-life insurance company has responsibility for a portfolio of $J = 1000$ policies. Assume further that the number of claims $\mathcal{N}$ is Poisson distributed with intensity $\mu = 0.01$, and that the claim sizes Z_i follow a log-normal distribution with $\mathrm{E}(Z_i) = 2$ and $\mathrm{sd}(Z_i) = 1.0$.

a) Find the parameters ξ and σ of the log-normal distribution.

b) Implement an algorithm for simulating log-normal variables using the inverse of normal CDF. Find Monte Carlo estimator of the expected value and variance and compare with the theoretical vales. Is there any indication of error in your simulation? Compare your results using your algorithm with simulations using a method included with your programming language of choice. Remember to check parameterization.

c) Compute the 95% and 99% reserve for this portfolio.

d) Assume now that the insurance policy has a deductible $a = 0.5$ and maximum $b = 3.0$. Compute the 95% and 99% reserve in this case. Explain the results.

Problem 2. Consider a put option with time to maturity $T = 1$ and the underlying return R_1 following the log-normal model with volatility $\sigma\sqrt{T}$, and assume that the risk-free return is $r = 0.04$.

a) Assume that $r_g = 0.06$ and $\sigma_1 = 0.25$. Compute value of this option using the Black-Scholes formula.

b) Assume now σ_1 varies between $0.25, 0.3, 0.35$. Compute the value of this option using Monte Carlo method and compare the results. For $\sigma_1 = 0.25$ compare with the result from point a). Comment on the relation between σ_1 and the value of the option.

c) Assume now that $\sigma_1 = 0.25$ and that there is another asset R_2 with $\sigma_2 = 0.35$ and that processes driving the two assets are correlated. For values of $\rho = -0.9, -0.5, 0.0, 0.5, 0.9$ and pay-off $X = \max(R_1 - R_2, 0)$ find the premium of the put option in this case. Comment on the results.

Good luck!