Let $X_1,...,X_n$ be mutually independent random variables with mgfs $M_{X_i}(t),\ i=1,...,n,$ and let $a_1,...,a_n$ and $b_1,...,b_n$ be constants. Then the mgf of $Z=\sum_{i=1}^{n}(a_1X_1+b_i)$ is

$$M_Z(t)=\exp\left(t\sum b_i\right)\prod_{i=1}^{n}M_{X_i}(a_i t)$$

This result may be used to prove that if $X_1,...,X_n$ are independent and $X_i \sim n(\mu_i,\sigma_i^2)$, then

$$Z=\sum_{i=1}^{n}(a_1X_1+b_i)\sim n\left(\sum a_i\mu_i+b_i\ ,\ \sum a_i^2\sigma_i^2\ \right)$$