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1.2.2 The Calculus of Probabilities

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1.2.4 Enumerating Outcomes

1.3 Conditional Probability and Independence

1.4 Random Variables

1.5 Distribution Functions

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