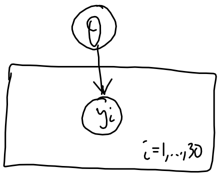
Half-time of a new drug

A medical trial for investigating the mean half-time θ of a new drug. The half-time y_i for $n=30$ patients were measured, $y=(y_1,\dots,y_n)^T$.

Before the measurements were done for these 30 patients, pre-expiriments indicated that the mean half-time θ is around $\mu_0 = 9$ hours with a standard deviation of $\tau_0 = 0.51$ hours, and with a relatively symmetric distribution. This is <u>prior</u> information (prior to obtaining any data).

The full model: $y_i \sim N(\theta, \sigma^2)$, $i=1,\dots,30$, we can assume $\sigma = 1.5$

$\theta \sim N(\mu_0, \tau_0^2)$



$p(\theta|y)$?

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We are going to show that for this simple example

$$p(\theta|y) = \frac{p(\theta)\cdot p(y|\theta)}{\underset{\substack{\uparrow \\ \text{constant w.r.t. } \theta}}{p(y)}} \propto N(\theta|\mu_0, \tau_0^2)\cdot \prod_{i=1}^{30} N(y_i|\theta, \sigma^2)$$

$$= N\left(\frac{\frac{1}{\tau_0^2}\mu_0 + \frac{n}{\sigma^2}\cdot \bar{y}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}} , \frac{1}{\frac{1}{\tau_0^2}+\frac{n}{\sigma^2}} \right)$$

If we did not have any prior knowledge on θ, we could say that (in this case)

$$p(\theta) \propto 1$$

Then $p(\theta|y) = N(\bar{y}, \frac{\sigma^2}{n})$

Classical 95% confidence interval for θ when we assume y_i iid $N(\theta, \sigma^2)$: $\bar{y} \pm 1.96 \cdot \frac{\sigma}{\sqrt{n}}$

which will yield numerically the same interval as the posterior interval for θ with $p(\theta) \propto 1$, BUT formally the interpretations are different!

More realistic: σ unknown: Classical inference: t-distribution
Bayesian inference: Include σ in θ and give it a prior

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