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10. Bayesian Computation

posterior $p(\theta|y) \propto \tilde{p}(\theta|y) = p(\theta)\, p(y|\theta)$

10.1. All Bayesian inference (almost..)

$\bar{h} = \int h(\theta) \quad p(\theta|y)\, d\theta$ (1)

- Estimation: $h(\theta) = \theta_1$
- uncertainty: $= (\theta_1 - \bar{\theta}_1)^2$
- prediction: $= p(y_{n+1}|\theta)$, i.e. $\underbrace{p(y_{n+1}|y_1 .. n)}_{=?} = \int p(y_{n+1}|\theta)\, \overbrace{p(\theta|y_1 .. y_n)}^{=?} d\theta$
- post. prob.: $p(A|y) = \int 1_A(\theta)\, p(\theta|y)\, d\theta$
  $h(\theta) = 1_A(\theta)$

Note: not all, e.g. posterior quantiles

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Simple Monte Carlo ("Simulation method")

$\bar{h} = \int h(\theta)\, p(\theta \quad)\, d\theta \approx \overbrace{\frac{1}{S}\sum_{s=1}^{S} h(\theta^s)}^{=\hat{h}_S} \xrightarrow[SLLN]{} \bar{h}$ (2)

$\theta^s \sim p(\theta \quad)$, i.i.d., $s = 1..S$

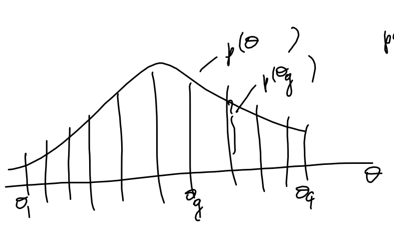
Problem, this replaces (1) by (2)

O'Hagan, "MC ~~MC~~ fundamentally unsound.." ~~Biometrika~~ Statistician (19~~xx~~ 87)

$(\hat{h}_S - \bar{h}) \big/ \frac{\sigma}{\sqrt{S}} \approx N(0,1)$, $\sigma^2 = Var(\ h(\theta)|y)$
CLT

10.3. Direct sim

& rejection sfg.



grid = seq(from=θ_1, to=θ_G, length=G)
pgrid = post(grid)
$\underline{\theta}^s$ = sample(grid, $\frac{1}{S}$, ~~p~~grid)

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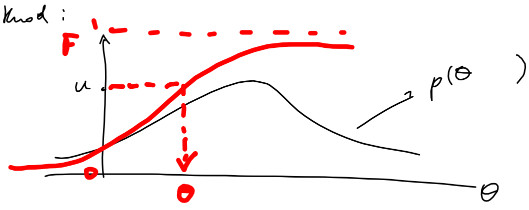
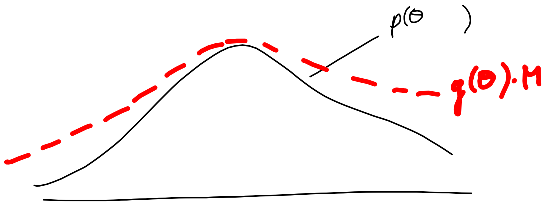
Inv. cdf method:



1. $u \sim \mathcal{U}(0,1)$
2. $\theta = F^{-1}(u)$

$\theta \sim p(\theta)$

Rejection method



(1) $\theta \sim g(\theta)$

(2) $w = \frac{p}{g} \cdot \frac{1}{M}$

(3) $u \sim \mathcal{U}(0,1)$

if $u \geq w$

$u < w \rightarrow$ return θ

$\theta \sim p(\theta)$

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10.4. Importance Spg.

(3)

$$\bar{h} = \int h(\theta) \underbrace{\frac{p(\theta)}{g(\theta)}}_{= w(\theta)} g(\theta)\, d\theta$$

$$= \int h(\theta)\, w(\theta)\, g(\theta)\, d\theta \approx \frac{1}{S} \sum h(\theta^s)\, w(\theta^s)$$ weight

$\theta^s \overset{iid}{\sim} g(\theta)$ "import. spl. density"

Probl: usually only have $\tilde{p}(\theta) = c \cdot p(\theta)$

$$c = \int 1 \frac{\tilde{p}(\theta)}{\tilde{g}} \tilde{g}\, d\theta$$

$$\boxed{\hat{h}_S = \frac{\sum w(\theta^s)\, h(\theta^s)}{\sum w(\theta^s) \cdot 1}}$$

$$w(\theta^s) = \frac{\tilde{p}}{g}$$

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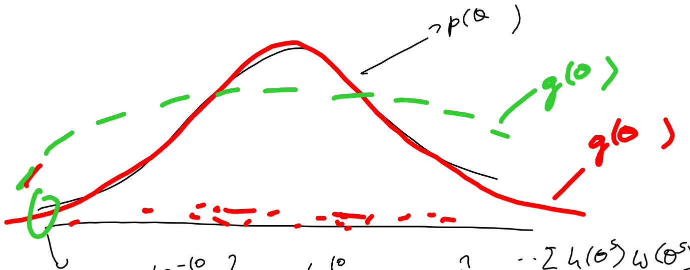

$$p = 10^{-10}, \quad q = 10^{-20} \Big\} \; w = 10^{10}$$

$$\hat{h} = \frac{\cdots \sum h(\theta^s)\, w(\theta^s)}{\cdots}$$

$\mathrm{Var}(\hat{h}_s)$ function of $\mathrm{Var}(w)$ (Geweke, 1989 Econometrica)

$S = 100$