Supplemental exercise 4

The joint posterior distr.: (with $a = 10^{-3}$, $b = 10^{-3}$)

$$\propto (\tau^2)^{-(a+1)} \exp\{-b/\tau^2\} \cdot \exp\left\{\frac{\mu^2}{2\cdot 10^6}\right\} \cdot \frac{1}{(\tau^2)^{J/2}} \exp\left\{-\frac{1}{2\tau^2}\sum_{j=1}^{J}(\theta_j-\mu)^2\right\} \cdot \exp\left\{-\frac{1}{2\cdot 10^5}\sum_{j=1}^{J}\beta_j^2\right\}$$

$$\cdot \prod_{j=1}^{J}\left[l_1(\beta_j)\cdot l_2(\beta_j,\theta_j)\right], \text{ der } l(\beta_j) = \left(\frac{\exp\{\beta_j\}}{1+\exp\{\beta_j\}}\right)^{y_j^c} \cdot \left(\frac{1}{1+\exp\{\beta_j\}}\right)^{n_j^c - y_j^c}$$

$$l(\beta_j,\theta_j) = \left(\frac{\exp\{\beta_j+\theta_j\}}{1+\exp\{\beta_j+\theta_j\}}\right)^{y_j^T} \cdot \left(\frac{1}{1+\exp\{\beta_j+\theta_j\}}\right)^{n_j^T - y_j^T}$$

Hence, the full cond. distr. are:

$$p(\tau^2 \mid \mu,\theta,\beta,y) = \text{Inv-Gamma}\left(a + \frac{J}{2},\ b + \frac{1}{2}\sum_{j=1}^{J}(\theta_j-\mu)^2\right)$$

$$p(\mu \mid \tau,\theta,\beta,y) = N\left(\frac{\frac{J}{\tau^2}\cdot\bar{\theta}}{10^{-6}+\frac{J}{\tau^2}},\ \frac{1}{10^{-6}+\frac{J}{\tau^2}}\right)$$

$$p(\theta_j \mid \mu,\theta_{(-j)},\beta,\tau,y) \propto \exp\left\{-\frac{(\theta_j-\mu)^2}{2\tau^2}\right\} \cdot \left(\frac{\exp\{\beta_j+\theta_j\}}{1+\exp\{\beta_j+\theta_j\}}\right)^{y_j^T} \cdot \left(\frac{1}{1+\exp\{\beta_j+\theta_j\}}\right)^{n_j^T - y_j^T}$$

$$p(\beta_j \mid \mu,\theta,\beta_{(-j)},\tau,y) \propto \exp\left\{-\frac{\beta_j^2}{2\cdot 10^5}\right\} \cdot \left(\frac{\exp\{\beta_j\}}{1+\exp\{\beta_j\}}\right)^{y_j^c} \cdot \left(\frac{1}{1+\exp\{\beta_j\}}\right)^{n_j^c - y_j^c} \cdot \left(\frac{\exp\{\beta_j+\theta_j\}}{1+\exp\{\beta_j+\theta_j\}}\right)^{y_j^T} \cdot \left(\frac{1}{1+\exp\{\beta_j+\theta_j\}}\right)^{n_j^T - y_j^T}$$

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$$\log p(\theta_j \mid \mu,\tau^2,\beta,\theta_{(-j)},y) = C - \frac{(\theta_j-\mu)^2}{2\tau^2} + y_j^T(\beta_j+\theta_j) - n_j^T \log(1+\exp\{\beta_j+\theta_j\}),\ j=1,\dots,J$$

$$\log p(\beta_j \mid \mu,\tau^2,\beta_{(-j)},\theta,y) = C - \frac{\beta_j^2}{2\cdot 10^5} + y_j^c\beta_j - n_j^c\log(1+\exp\{\beta_j\})$$
$$+ y_j^T(\beta_j+\theta_j) - n_j^T\log(1+\exp\{\beta_j+\theta_j\}),\ j=1,\dots,J$$

Metropolis for θ_j in one step of the Gibbs sampler: Target distr.: $p(\theta_j \mid \mu,\tau^2,\theta_{(-j)},\beta,y)$

Proposal distribution (jumping distribution): $J_{t,\theta_j}(\theta_j^* \mid \theta_j^{t-1}) = N(\theta_j^{t-1},\ c_j^2)$

Compute
$$r = \exp\left\{\log\left\{\frac{p(\theta_j^* \mid \mu^{(t-1)},\tau^{2(t-1)},\beta^{(t-1)},\theta_{(-j)}^{(t-1)},y)}{p(\theta_j^{t-1} \mid \mu^{(t-1)},\tau^{2(t-1)},\beta^{(t-1)},\theta_{(-j)}^{(t-1)},y)}\right\}\right\} = \exp\{\log p(\theta_j^* \mid \dots) - \log p(\theta_j^{t-1} \mid \dots)\}$$

Acceptance probability: $\alpha(\theta_j^{t-1},\theta_j^*) = \min\{r, 1\}$

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