UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in:	STK9021 — Applied Bayesian Analysis and Numerical Methods
Day of examination:	Tuesday 13 December
Examination hours:	14.30 – 18.30

This problem set consists of 2 pages.

Appendices:	Some useful formulas. List of distributions
Permitted aids:	Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

a

Suppose we have n independent and identically distributed observations $y = (y_1, \ldots, y_n)$, with

$$\begin{aligned} y_i &\sim \text{Poisson}(\theta),\ i = 1, \ldots, n \\ \theta &\sim \text{Gamma}(\alpha, \beta) \end{aligned}$$

(i) Find the posterior distribution for θ for $n = 1$, hence conditional on data $y = y_1$

(ii) Then find the posterior distribution for θ for the general case of $n \geq 1$, hence conditional on data $y = (y_1, \ldots, y_n)$.

b

Consider the number of pregnant women arriving at a particular hospital during July in a particular year, denoted by y. Suppose $y_1, \ldots, y_5$ are the numbers of pregnant women arriving at the hospital during July the last 5 years. We can assume that $y_1, \ldots, y_5$ are independent and identically Poisson distributed with parameter θ. The history for July prior to these last 5 years suggests that an appropriate prior distribution for θ is $\text{Gamma}(5, \frac{1}{6})$.

(i) Find the posterior mean for θ given this model and data $(y_1, \ldots, y_5) = (67, 43, 74, 37, 59)$.

(ii) For estimating θ, what loss function does the estimate from (i) correspond to?

(iii) Find the posterior variance for θ.

(Continued on page 2.)

c

The posterior predictive distribution of how many pregnant women $\tilde{y}$ will arrive at the hospital during July next year is of particular interest. There is no information that distinguishes this year from the previous 5 years, and conditional on θ, $\tilde{y}$ is independent of $y_1, \ldots, y_5$.

(i) Show that the posterior predictive distribution for $\tilde{y}$ is Negative-binomial.

(ii) Report the posterior predictive mean and variance for $\tilde{y}$ given the data from b.

Problem 2

Suppose $y_1, \ldots, y_n$ are independent and identically distributed observations with

$$y_i \sim N(\mu, \sigma^2), \; i = 1, \ldots, n$$
$$\mu \sim N(\mu_0, \tau^2)$$

where σ^2, μ_0, τ^2 are fixed, known quantities.

a

Derive the posterior distribution for μ. In particular, state the posterior mean and variance of μ.

b

(i) Show that the posterior predictive distribution for $\tilde{y}$ is Normal.

(ii) Find the posterior predictive mean and variance of $\tilde{y}$.

THE END