UiO : Matematisk institutt

Det matematisk-naturvitenskapelige fakultet

STK-4051/9051 Computational Statistics Spring 2020
Comments to exercise 8

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Ex 26

a) See solution, it is nice to know that the distribution is stationary.
b) Solution is a bit brief, but see also 27a) to get the expression for the likelihood
c) Too ease the understanding of the derivation, think of this as a proof by induction, the lines here is the general step from t-1 to t. Use also the expression in 26b to derive the result
d) See code,
e) The comment in the solution, is good.

If you have completed this task you have done a Sequential Monte Carlo, perhaps for the first time. Good Job ☺

Ex 27

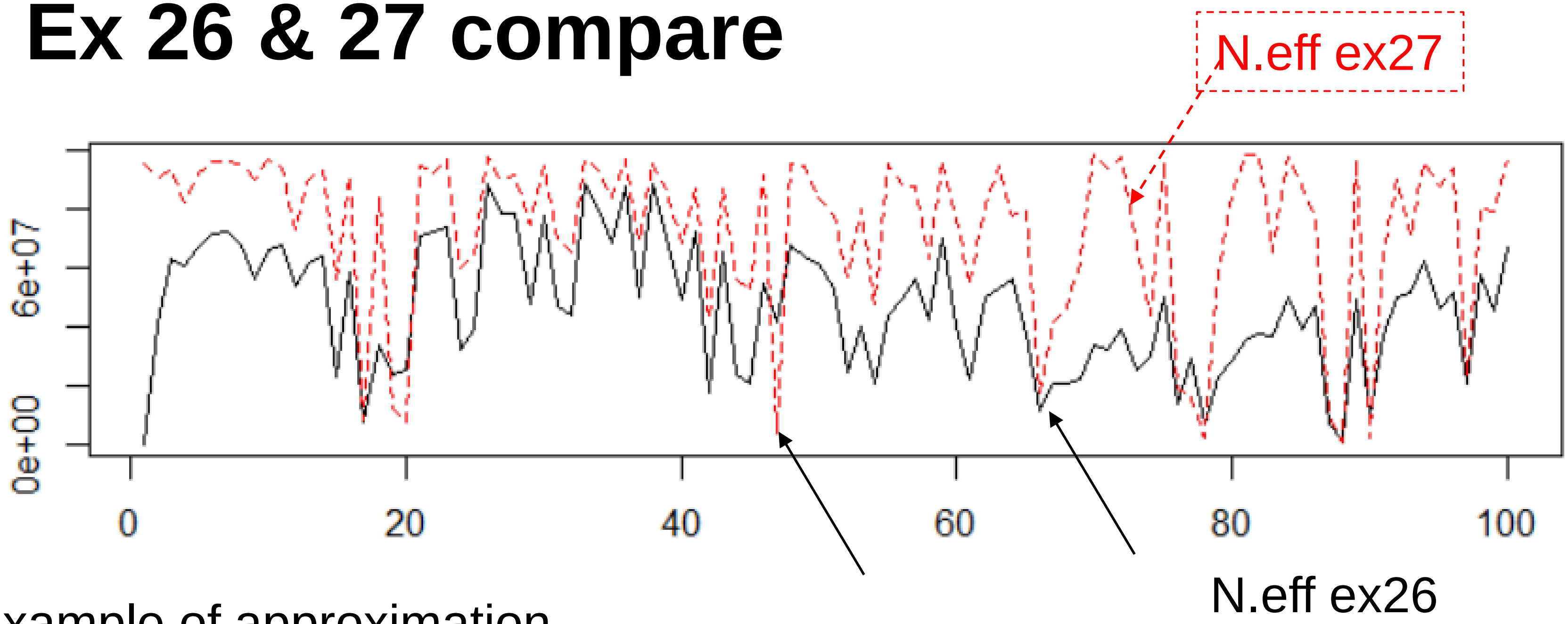
a) Since y is fixed, we can get rid of y!

b) Lots of computations here. In principle it is just a taylor expansion. The details are a bit messy….

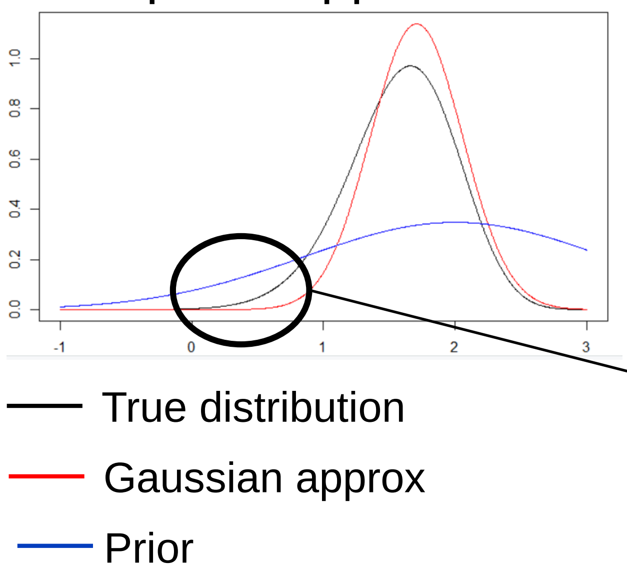
c) It is allways a challenge to program complex expressions, check the implementation twice (or more).

d) The upside is the effective number of samples increases with more than 50% on average

Ex 26 & 27 compare



Example of approximation



The approximation (ex 27) is not allways better

The main problem is the approximation in the left tail. In this region Gauss approx. is too low which gives large weights

Ex 29

- This is for you to get a feeling with the different methods. So not much to say here except that you should try it out.

- Note on the lambda for control variates:
  - We can compute this ratio using the input variables:
  - That is compute the variance and covariance of: $h(X_i)$ and $c(Y_i)$
  - $\lambda = -\text{cov}(h(X), c(Y))/\text{var}(c(Y))$
  - Remember that this number was quite robust towards small deviations

$$\hat{\mu}_{MC} = N^{-1} \sum_{i=1}^{N} h(\mathbf{X}_i)$$

$$\hat{\theta}_{MC} = N^{-1} \sum_{i=1}^{N} c(\mathbf{Y}_i)$$

$$\lambda = -\frac{\text{cov}[\hat{\mu}_{MC}, \hat{\theta}_{MC}]}{\text{var}[\hat{\theta}_{MC}]}$$

Ex 30

- Run the test example.
- The use of correlated samples gives an 10 times improvement of the standard deviation
- This corresponds to the commonly known difference between a paired test and a two sample test.

Ex 6.4

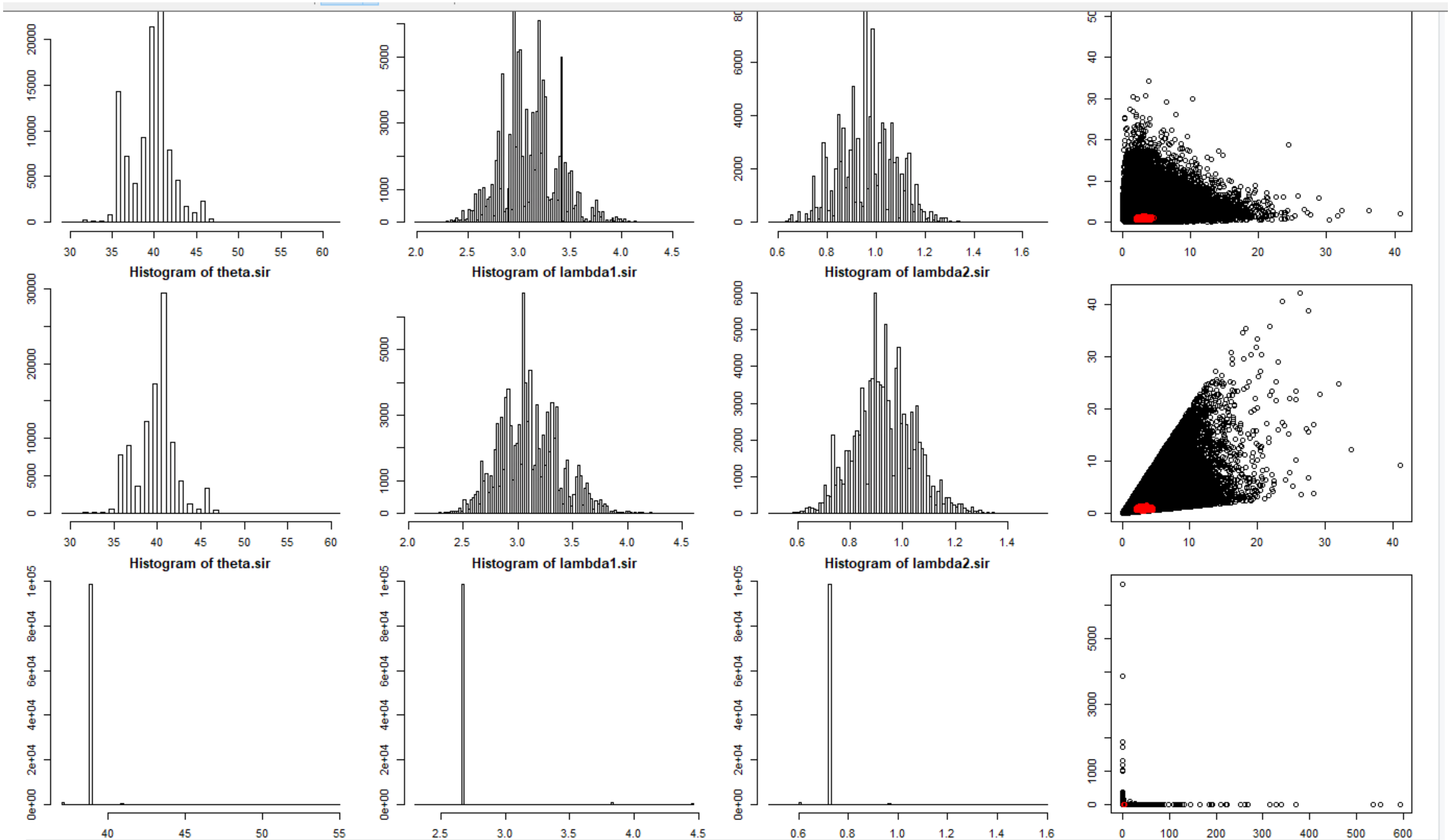
- See the code for all three examples.
  - Note that frequently we are in the situation that we have many products of numbers. Ant then vi divide by a product of some other numbers. In these cases. It is always recommended to working on the log-scale as this is more stable

$$\frac{\prod_{i=2}^{n} f(x_i|x_{i-1})}{\prod_{i=2}^{n} g(x_i|x_{i-1})} = \exp\left\{\sum_{i=2}^{n} \log(f(x_i|x_{i-1})) - \log(g(x_i|x_{i-1}))\right\}$$

Often give
numerical problems
e.g. (0/0) or Nan/Nan

Better

Ex 6.4


Histogram of theta.sir
Histogram of lambda1.sir
Histogram of lambda2.sir
Histogram of theta.sir
Histogram of lambda1.sir
Histogram of lambda2.sir

Ex 6.4

- In the posterior of a and b
  - The marginal distributions are similar
  - Joint distribution of lambda 1 and 2 are different
- The estimates are robust towards the formulation of prior distribution

```
[1] "Prior from (a)"
Estimate of theta 39.78553
Credibility interval for theta: 36 46
Estimate of lambda1 3.105188
Credibility interval for lambda1: 2.566814 3.742627
Estimate of lambda2 0.9649861
Credibility interval for lambda2: 0.7483087 1.186242
[1] "Prior from (b)"
Estimate of theta 40.08188
Credibility interval for theta: 36 46
Estimate of lambda1 3.106418
Credibility interval for lambda1: 2.605762 3.703612
Estimate of lambda2 0.9308218
Credibility interval for lambda2: 0.7120824 1.167468
[1] "Prior from (c)"
Estimate of theta 38.99738
Credibility interval for theta: 39 39
Estimate of lambda1 2.677348
Credibility interval for lambda1: 2.662579 2.662579
Estimate of lambda2 0.7272437
Credibility interval for lambda2: 0.7268339 0.7268339
Effective number of samples a: 86.77606  b: 191.9267  c: 1.021563W
```

- In c. The method is a failure, we need many more samples to get this right
- A too wide prior is sometimes not helpful

Important to know when the method has failed!