UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: STK4060 — Time Series

Day of examination: Monday June 4'th 2012

Examination hours: 09.00 – 13.00

This problem set consists of 2 pages.

Appendices: None

Permitted aids: Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

Consider a linear time series $\{x_t\}_{t=-\infty}^{\infty}$ of the form

$$x_t = \mu + \sum_{j=-\infty}^{\infty} \psi_j w_{t-j}$$

where $\sum_{j=-\infty}^{\infty} |\psi_j| < \infty$ and $\{w_t\}_{t=-\infty}^{\infty}$ is a sequence of white noise, i.e a sequence of random variables where $E[w_t] = 0$, $E[w_t^2] = \sigma_w^2$ for all t and $E[w_s w_t] = 0$ for all $s \neq t$.

a) Explain why $\{x_t\}_{t=-\infty}^{\infty}$ is weakly stationary. Define the autocovariances $\gamma(h)$ of $\{x_t\}_{t=-\infty}^{\infty}$ and show that

$$\gamma(h) = \sigma_w^2 \sum_{j=-\infty}^{\infty} \psi_j \psi_{j+h},\ h = 0, \pm 1, \pm 2, \ldots.$$

b) Explain why a weakly stationary autoregressive time series of order 1 having finite variance and satisfying the stochastic difference equation

$$x_t = \phi x_{t-1} + w_t,\ t = 0, \pm 1, \pm 2, \ldots$$

where $|\phi| < 1$, is a linear process. What does it mean that it is causal?

c) What is meant by a causal, invertible ARMA-process? How can properties of their autocorrelation (ACF) and partial autocorrelaton functions (PACF) be used to identify such processes?

d) Define the Yule-Walker equations, and explain how they can be used to estimate the coefficients in a stationary, causal autoregressive process of order p, i.e. an AR(p) process.

(Continued on page 2.)

e) Consider the stationary, causal autoregressive process of order 2

$$x_t = (5/6)x_{t-1} - (1/6)x_{t-2} + w_t.$$

Find the autocorrelation (ACF) and partial autocorrelation function (PACF) of this process.

Problem 2

Consider a stationary time series $\{x_t\}_{t=-\infty}^{\infty}$ satisfying $\sum_{h=-\infty}^{\infty} |\gamma(h)| < \infty$.

a) Explain how the spectral density is defined and how it should be interpreted.

b) What is the spectral density in the autoregressive time series

$$x_t = 0.5x_{t-1} + w_t, t = 0, \ \pm 1, \pm 2, \dots$$

where $\{w_t\}_{t=-\infty}^{\infty}$ is a sequence of white noise, $wn(0, \sigma_w^2)$.

c) If $x_1, \dots, x_n$ are observations from the time series $\{x_t\}_{t=-\infty}^{\infty}$ defined in the beginning of the problem, how are the periodogram $I(j/n), j = 0, 1, \dots, n-1$ defined? Show that the expectation satisfies

$$E[I(j/n)] = \sum_{h=-(n-1)}^{(n-1)} (1 - \frac{|h|}{n})\gamma(h)e^{-2\pi i(j/n)h}, \ j = 1, \dots, n-1$$

d) Describe the main properties of the distribution of the periodogram, and explain how they can used to estimate the spectral density.

END