UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: STK9060 — Time series

Day of examination: Monday June 6th 2016

Examination hours: 14.30 – 18.30

This problem set consists of 2 pages.

Appendices: Note on state space model and Kalman filter

Permitted aids: Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

Let x_t be a stationary time series with $E[x_t] = 0$ and $var(x_t) < \infty$.

a) Explain what an ARMA(p,q) time series is and what it means that is causal and invertible.

b) Is the time series x_t satisfying

$$x_t = -0.1x_{t-1} + 0.12x_{t-2} + w_t$$

causal?

c) Explain what the partial autocorrelation coefficient is and how it is used to determine the order of an autoregressive time series.

Problem 2

The bivariate AR(1) process

$$\begin{pmatrix} \psi_t \\ \psi_t^* \end{pmatrix} = 0.5 \begin{pmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{pmatrix} \begin{pmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{pmatrix} + \begin{pmatrix} \kappa_t \\ \kappa_t^* \end{pmatrix}$$

where κ_t and κ_t^* are independent $N(0, \sigma_\kappa^2)$ and $N(0, \sigma_{\kappa^*}^2)$ respectively can be used for describing cyclical patterns. Consider first λ as known. Then two ways to formulate the model as a part of a larger model is as

(i) the *trend and cycle* model where

$$\begin{aligned} y_t &= \mu_t + \psi_t + \epsilon_t \\ \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t \\ \beta_t &= \beta_{t-1} + \zeta_t \end{aligned}$$

(Continued on page 2.)

and as

(ii) the *cyclical trend* model where

$$\begin{aligned} y_t &= \mu_t + \epsilon_t \\ \mu_t &= \mu_{t-1} + \beta_{t-1} + \psi_{t-1} + \eta_t \\ \beta_t &= \beta_{t-1} + \zeta_t \end{aligned}$$

where ϵ_t, η_t and ζ_t are independent and mutual independent $N(0, \sigma_\epsilon^2)$, $N(0, \sigma_\eta^2)$ and $N(0, \sigma_\zeta^2)$ variables, which also are independent of the κ_t's and κ_t^*'s.

a) Formulate both models as state-space models.

b) Assume that the initial state has expectation zero and covariance matrix equal to the identity matrix so $\mathbf{x}_0^0 = 0$ and $P_0^0 = I$. Describe what the Kalman gain at time 1, K_1, is for the *trend and cycle* model.

c) If now the parameter λ is not assumed to be known, how would you estimate it based on observations $y_1, \ldots, y_n$?

d) How can the results in part a), b) and c) be interpreted if ϵ_t, η_t, ζ_t, κ_t and κ_t^* are only assumed to be uncorrelated white noise, i.e. $\epsilon_t \sim wn(0, \sigma_\epsilon^2)$, $\eta_t \sim wn(0, \sigma_\eta^2)$ and $\zeta_t \sim wn(0, \sigma_\zeta^2)$, $\kappa_t \sim wn(0, \sigma_\kappa^2)$ and $\kappa_t^* \sim wn(0, \sigma_{\kappa^*}^2)$?

Problem 3

Consider observations $z_1, \ldots, z_n$ from a time series with covariance function $\gamma(h)$ satisfying $\sum_{-\infty}^{\infty} |\gamma(h)| < \infty$ so that the spectral density $f_z(\omega)$ exists. Recall that the discrete Fourier transform is given as $d_z(j/n) = \frac{1}{\sqrt{n}} \sum_{t=1}^{n} z_t e^{-2\pi i (j/n)t}$, $j = 0, \ldots, n-1$.

a) Define the periodogram and state its main properties. Explain how the periodogram can be used for estimating the spectral density.

b) Let the time series y_t be defined as

$$y_t = \mu + x_t, \ t = 1, \ldots, n$$

where x_t is a stationary time series with discrete Fourier transform d_x. The expectation $E[x_t] = 0$ and μ is a known constant. Express the discrete Fourier transform of y_t in terms of the discrete Fourier transform of x_t, i.e. d_x, and μ.

c) Let y_t be as in 3b) and let

$$y'_t = \begin{cases} y_t, \ t = 1, \ldots, n \\ 0, \ t = n+1, \ldots, n' \end{cases}$$

i.e. y_t padded. Find the discrete Fourier transform of y'_t. Compare the result to what you found in part b).

d) Discuss whether it is preferable to subtract μ from the series y_t or from the padded version y'_t when the focus of interest is the periodic behavior of y_t.

END