UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: STK9060 — Time Series

Day of examination: Wednesday May 28th 2014

Examination hours: 09.00 – 13.00

This problem set consists of 2 pages.

Appendices: None

Permitted aids: Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

In this problem you are asked to consider autoregressive time series.

a) How is an autoregressive time series of order p defined? Explain what it means that it is causal.

Consider the autoregressive time series of order two

$$x_t = x_{t-1}/3 + 2x_{t-2}/9 + w_t$$

where w_t is a time series of white noise, i.e. $w_t \sim wn(0, \sigma^2)$.

b) Show that x_t defined above is causal and find the representation $x_t = \sum_{j=-\infty}^{\infty} \psi_j w_{t-j}$ where $\psi_0 = 1$.

c) Explain that the auto correlation function of x_t satisfies a difference equation. Find the the auto correlation function of x_t.

d) What is the partial autocorrelation function of x_t?

Problem 2

Consider observations $y_1, \ldots, y_n$ described by the equations

$$\begin{aligned} x_t &= \mu + a x_{t-1} + w_t,\ |a| < 1 \\ y_t &= b x_t + v_t \end{aligned}$$

where $w_t, t = 1, \ldots, n$ and $v_t, t = 1, \ldots, n$ are independent random variables where $w_t \sim iidN(0, \sigma_w^2)$ and $v_t \sim iidN(0, \sigma_v^2)$. The variable x_0, $x_0 \sim N(0, \sigma_0^2)$, is independent of w_t and v_t

a) Interpret the equations as a state-space model. Explain what the state equation and the observation equation are.

(Continued on page 2.)

b) What are the conditions for the bivariate series $(x_t, y_t)'$ to be stationary? Find the expectation and covariance of the stationary series $(x_t, y_t)'$ satisfying the equations above?

c) What is the stationary distribution of $(x_t, y_t)'$ described in part b)?

Problem 3

Let $x_t, t = 0, \pm 1, \pm 2, \ldots$ be a stationary time series such that $\sum_{h=-\infty}^{\infty} |\gamma(h)| < \infty$ where $\gamma(h)$ is the covariance between x_{t+h} and x_t.

a) Define the spectral density $f_X(\omega)$ of x_t. Explain why $f_X(\omega) = f_X(-\omega)$ and $f_X(\omega) = f_X(1 - \omega)$.

Let $a_j, j = 0, \pm 1, \pm 2, \ldots$ be a sequence of scalars so that $\sum_{j=-\infty}^{\infty} |a_j| < \infty$. Let $y_t = \sum_{j=-\infty}^{\infty} a_j x_{t-j}$.

b) Show that the spectral density of y_t has the form

$$f_Y(\omega) = |A(\omega)|^2 f_X(\omega)$$

where $A(\omega) = \sum_{j=-\infty}^{\infty} a_j \mathrm{e}^{-2\pi i \omega j}$.

c) Let w_t be a white noise series, $w_t \sim wn(0, \sigma_w^2)$, and y_t be the ARMA(p,q) process defined by $\phi(B) y_t = \theta(B) w_t$ where the roots of the polynomials $\phi(z) = 0$ and $\theta(z) = 0$ are outside the unit circle. Use the result from part b) to explain that

$$f_Y(\omega) = \sigma_w^2 \frac{|\theta(\mathrm{e}^{-2\pi i \omega})|^2}{|\phi(\mathrm{e}^{-2\pi i \omega})|^2}.$$

END