# SOLUTIONS TO EXERCISES WEEK 41

## Exercise 3.10

$N_h(t)$ counting processes with intensity processes $\lambda_h(t) = \alpha_h(t) Y_h(t)$ for $h = 1, 2$.

Under $H_0: \alpha_1(t) = \alpha_2(t)$ the aggregated counting process $N_\cdot(t) = N_1(t) + N_2(t)$ has intensity process $\lambda_\cdot(t) = \alpha(t) Y_\cdot(t)$ where $Y_\cdot(t) = Y_1(t) + Y_2(t)$ and $\alpha(t)$ is the common value of $\alpha_1(t)$ and $\alpha_2(t)$ under $H_0$.

We know from (3.54) that the test statistic $Z_1(t_0)$ has predictable variation process

$$\langle Z_1 \rangle (t_0) = \int_0^{t_0} \frac{L^2(t)\, Y_\cdot(t)}{Y_1(t)\, Y_2(t)} \alpha(t)\, dt$$

under $H_0$. The variance estimator (3.55) takes the form

$$V_{11}(t_0) = \int_0^{t_0} \frac{L^2(t)}{Y_1(t)Y_2(t)} \, dN_\cdot(t)$$

By using the decomposition (under $H_0$) $dN_\cdot(t) = \alpha(t) Y_\cdot(t) dt + dM_\cdot(t)$ we get under $H_0$:

$$V_{11}(t_0) = \int_0^{t_0} \frac{L^2(t)}{Y_1(t)Y_2(t)} \left( \alpha(t) Y_\cdot(t) dt + M_\cdot(t) \right)$$

$$= \langle Z_1 \rangle (t_0) + \int_0^{t_0} \frac{L^2(t)}{Y_1(t) Y_2(t)} \, dM_\cdot(t)$$

Here the latter term on the right hand side is a mean zero martingale, and hence

$$E V_{11}(t_0) = E \langle Z_1 \rangle (t_0)$$

But we know that

$$\operatorname{Var} Z_1(t_0) = E \langle Z_1 \rangle (t_0)$$

and hence $V_{11}(t_0)$ is unbiased under $H_0$.

## Exercise 3.4

We know from (3.53) in the ABG-book that $Z_1(t_0)$ is a mean zero martingale under the null hypothesis.

Hence we have $E\{Z_1(t_0)\} = 0$.

Considering the logrank test, we have that [cf formula just above (3.60)]

$$Z_1(t_0) = N_1(t_0) - E_1(t_0)$$

Hence we have under $H_0$ that

$$E\{N_1(t_0) - E_1(t_0)\} = 0$$

so that

$$E\{E_1(t_0)\} = E\{N_1(t_0)\}$$