Additional exercise 1 Show the identity

$$\frac{\hat{S}(t)}{S^\star(t)} - 1 = -\int_0^t \frac{\hat{S}(s-)}{S^\star(s)} \frac{J(s)}{Y(s)} dM(s) \tag{1}$$

where $\hat{S}(t)$ is the Kaplan-Meier estimator for the survival function $S(t)$, $J(t) = I(Y(t) > 0$, $S^\star(t) = \exp(-\int_0^t J(s)\alpha(s)ds$ and notation otherwise as in the lectures and book.

The equation is a special case of Duhamel's equation (See Appendix A.1 of ABG, equation (A.12) on pg. 461).

Hint: It is sufficient to show that

$$\frac{\hat{S}(t-)}{S^\star(t-)} - \frac{\hat{S}(t)}{S^\star(t)} = -\frac{\hat{S}(t-)}{S^\star(t)} \frac{J(t)}{Y(t)} dM(t).$$