Additional exercise 2 Show that the estimated transition probabilities in equations (3.69) and (3.70) in ABG for the competing risk situation, in the lectures alternatively written as

$$\widehat{\mathrm{P}}_{00}(s,t) = \prod_{s<u\leq t}\left[1 - \frac{dN_0(u)}{Y_0(u)}\right],$$

and

$$\hat{P}_{0h}(s,t) = \int_s^t \hat{P}_{00}(s,u-)\frac{dN_{0h}(u)}{Y_0(u)}.$$

are special cases of the Aalen-Johansen estimator (3.80).

Hints: (a) Note that it is sufficient to show this when $s = 0$. (b) Argue recursively in the sense that you show the relation for $t = 0$ and then for t_{k+1} given that it holds for t_k, i.e. the previous event time. (c) Also note that you formally need to introduce all transition probabilities $\hat{P}_{jk}(s,t)$ although those not given will equal zero or one for all (s,t).

Additional exercise 3 Verify that the estimated transition probabilities in ABG, eq. (3.74)-(3.76), given by slightly different expressions in the lectures, are special cases of the Aalen-Johansen estimator (3.80).