Solution to 11 f)

In the analysis we then have: $\tilde{\varepsilon}_i = \beta_p x_{ip} + \varepsilon_i$ where we assume that ε_i is independent of x_{ip}, and have from the text that,

$$\text{Cov}(\varepsilon_i, \varepsilon_j) = \sigma^2 I(i = j)$$

$$Cov\left(x_p(\boldsymbol{s}_i), x_p(\boldsymbol{s}_j)\right) = C_p(\|\boldsymbol{s}_i - \boldsymbol{s}_j\|)$$

Where we have used the notation $x_{ip} = x_p(\boldsymbol{s}_i)$, to emphasis the spatial characteristic of covariate number p.

$$Cov(\tilde{\varepsilon}_i, \tilde{\varepsilon}_j) = \sigma^2 I(i = j) + \beta_p^2 \cdot C_p(\|\boldsymbol{s}_i - \boldsymbol{s}_j\|)$$

Assume that a process it is known to have strong dependence on a spatially variable parameter that we are not able to measure. By introducing an error term with spatial dependence we can still make inference with respect to other parameters in the model.

In a general case we may say that is possible to partially account for missing covariates by introducing dependencies in the residuals. If there is (very) long range dependencies in the residuals this might be a sign of a missing covariate.