Surface prediction using rejection sampling to handle non-linear relationships

Reducing structure uncertainty by using more information

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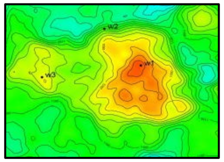
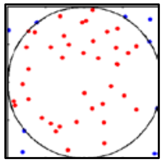
Surface prediction



Rejection sampling

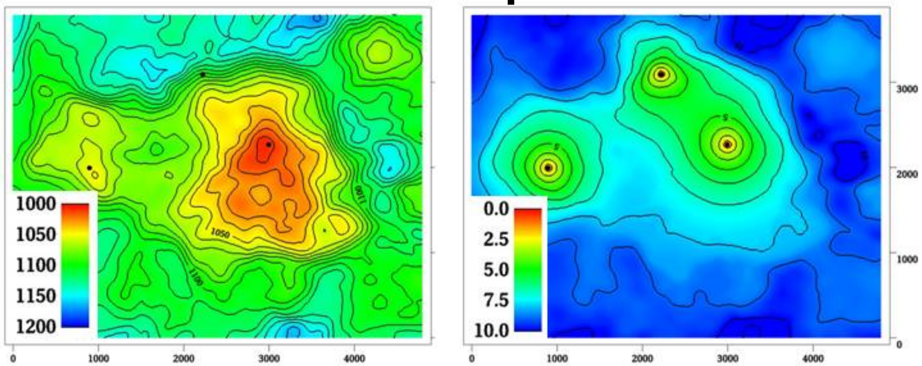


Spill points

Volumes

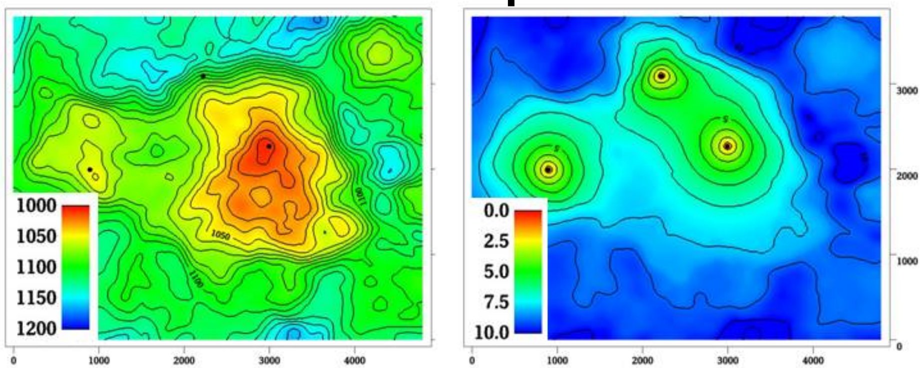
Surfaces

Prediction with prediction error:

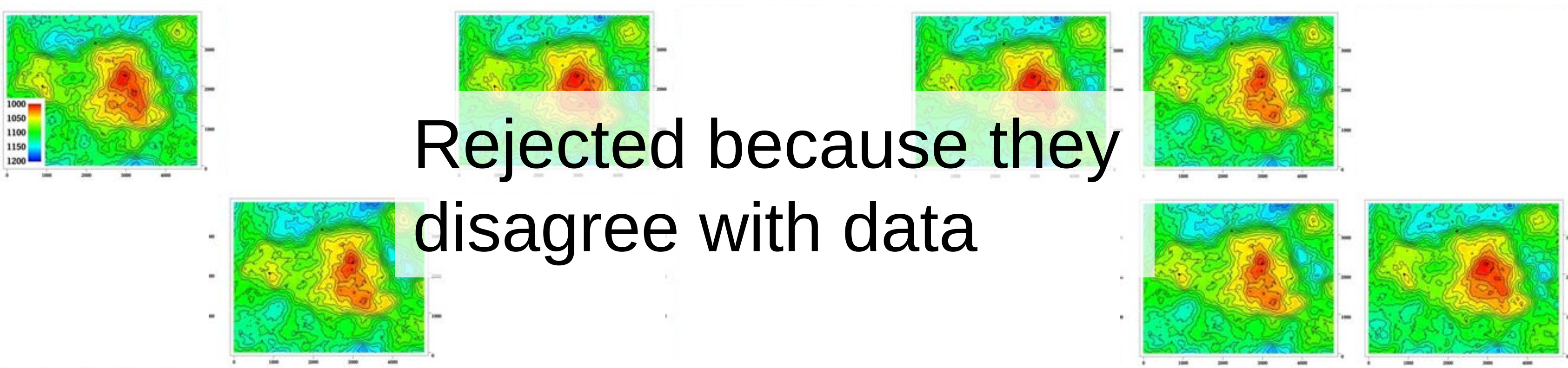


Surfaces

Prediction with prediction error:



Simulated realizations:

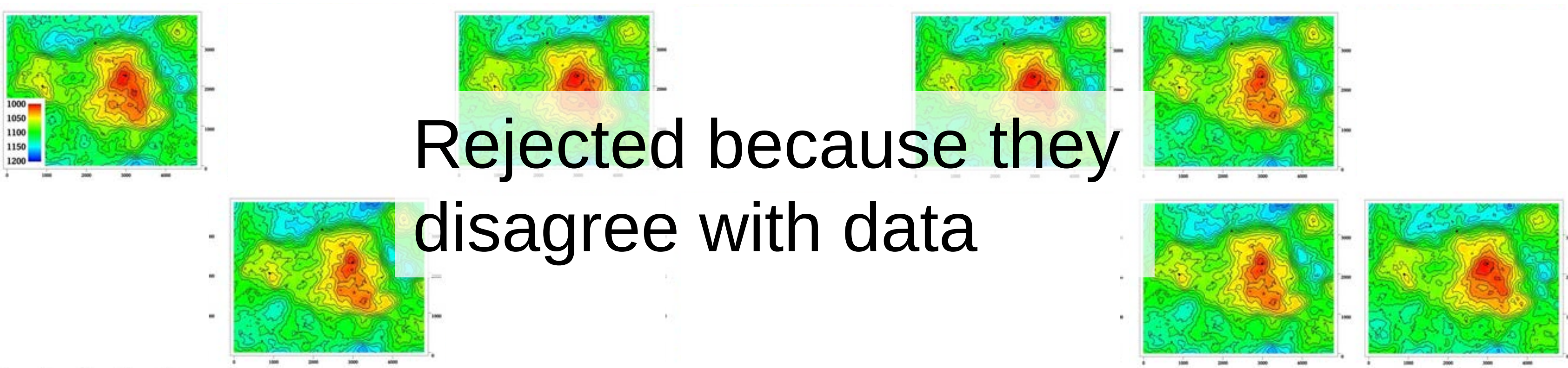


Rejected because they disagree with data

Rejection sampling

1. Disagree with spill point information
2. Disagree with horizontal wells

Simulated realizations:

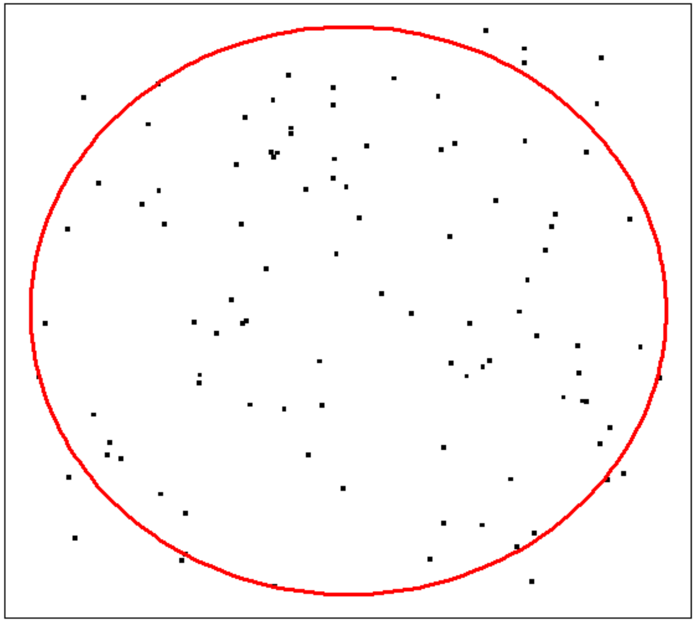


Rejected because they disagree with data

3.141 592 653 589 793 238 462 643…

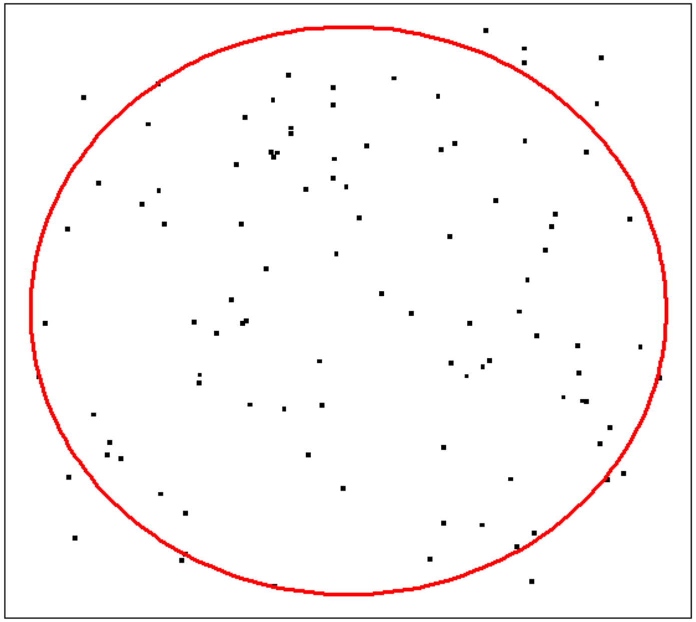
Lets find π using rejection sampling.

$$\pi = 4\,\frac{A_{circle}}{A_{square}}$$



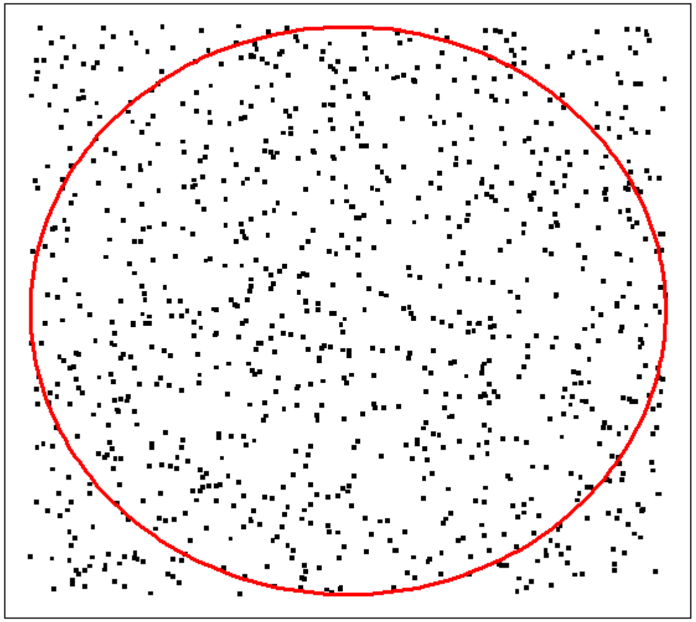
3.141 592 653 589 793 238 462 643…

# of samples	Estimate of π
100	3.4
1 000	
10 000	
100 000	
1 000 000	
10 000 000	



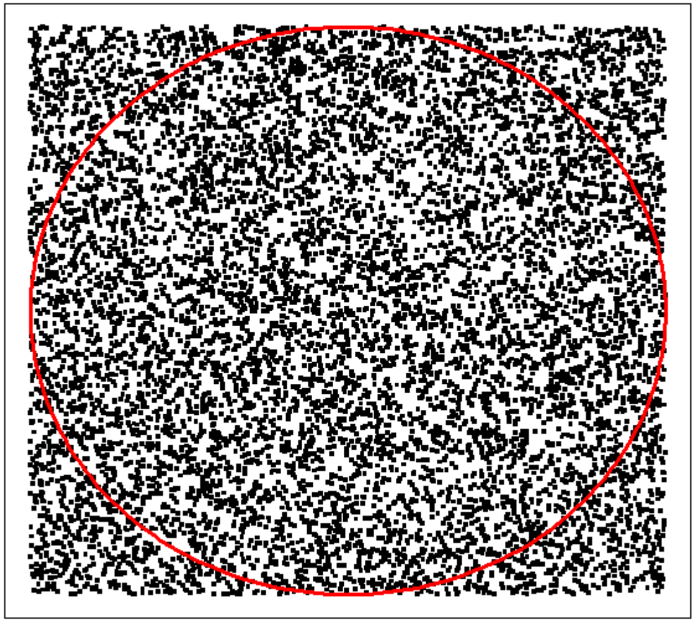
3.141 592 653 589 793 238 462 643…

# of samples	Estimate of π
100	3.4
1 000	3.16
10 000	
100 000	
1 000 000	
10 000 000	



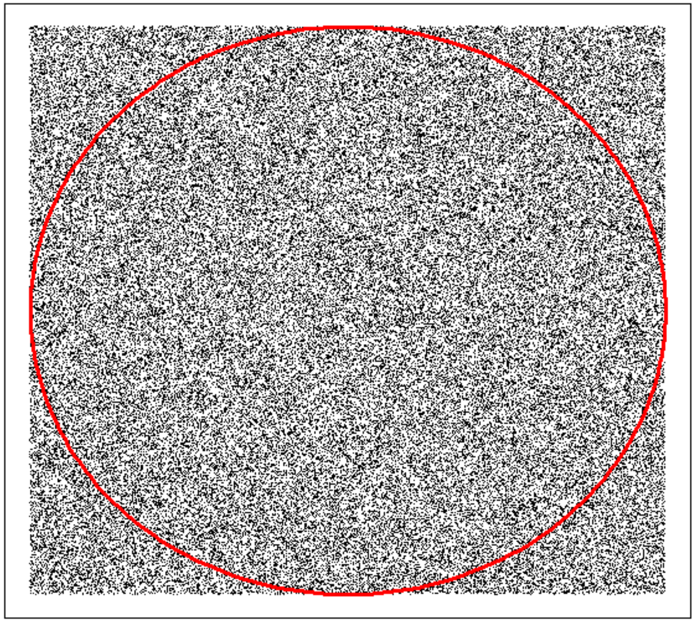
3.141 592 653 589 793 238 462 643…

# of samples	Estimate of π
100	3.4
1 000	3.16
10 000	3.11
100 000	
1 000 000	
10 000 000	



3.141 592 653 589 793 238 462 643…

# of samples	Estimate of π
100	3.4
1 000	3.16
10 000	3.11
100 000	3.13
1 000 000	
10 000 000	



3.141 592 653 589 793 238 462 643…

# of samples	Estimate of π
100	3.4
1 000	3.16
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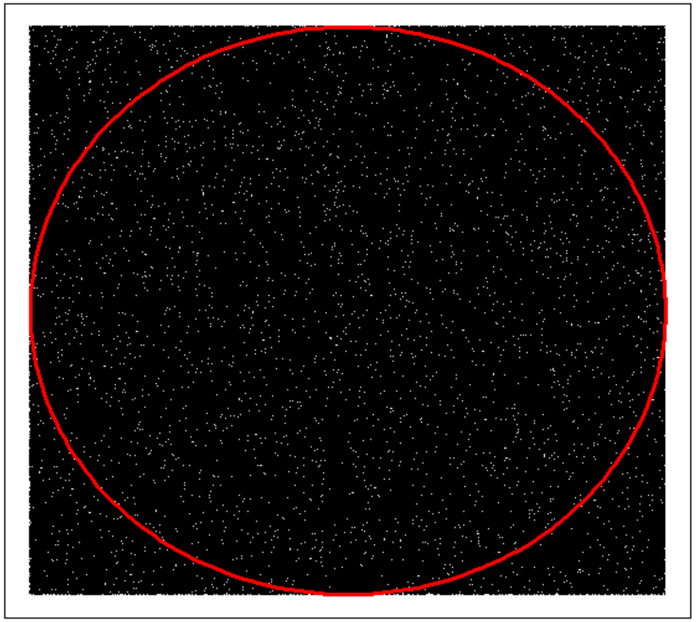
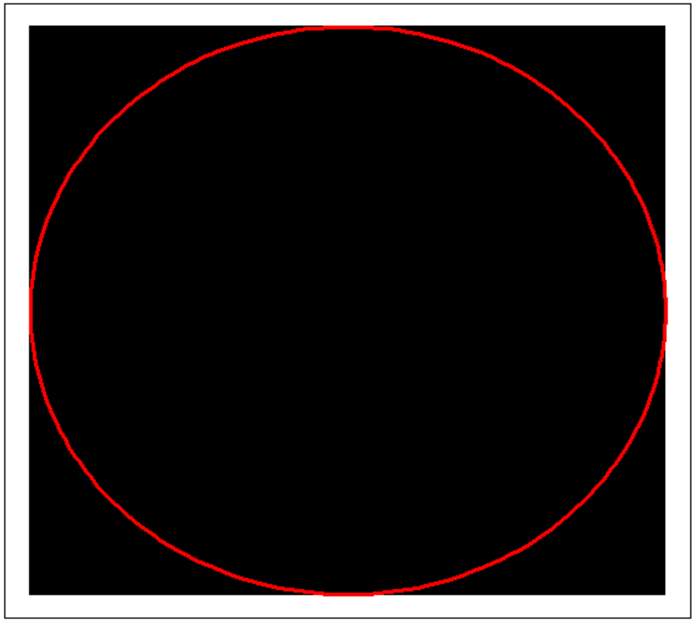


3.141 592 653 589 793 238 462 643…

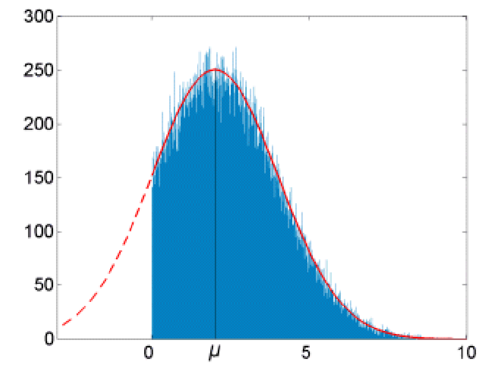
# of samples	Estimate of π
100	3.4
1 000	3.16
10 000	3.11
100 000	3.13
1 000 000	3.141
10 000 000	3.1414



Monte Carlo convergence is slow: $\sim 1/\sqrt{N}$

Rejection sampling

- General and simple
- Easy to draw samples when
  - $x_n \sim \mathrm{Prob}(x) = p(x)$ is simple to draw
  - Rejection criteria are simple: E.g $0 < x_n$



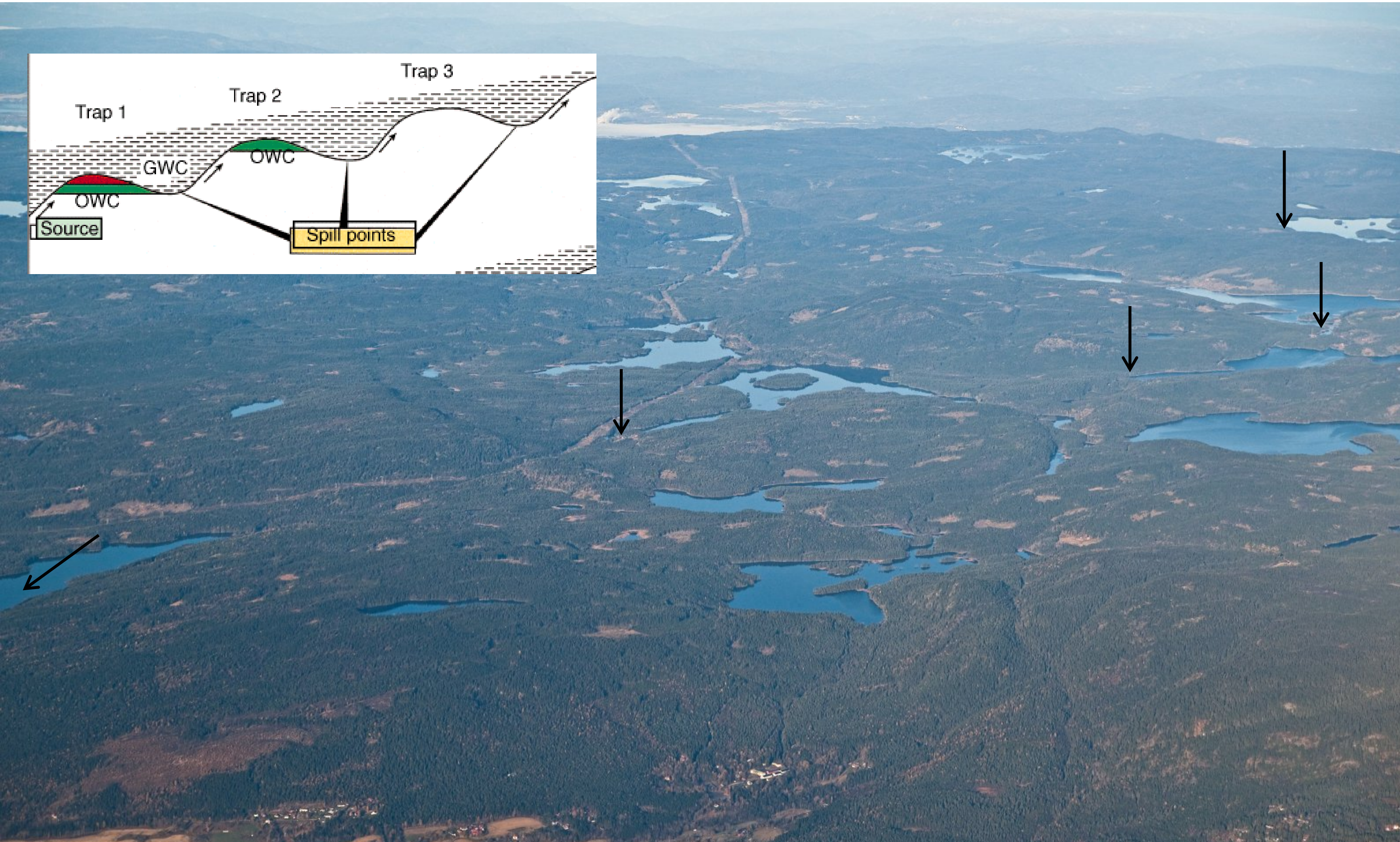
- Exact estimates:

$$A = \int_a^b f(x)\, p(x)\, dx = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} f(x_n)$$

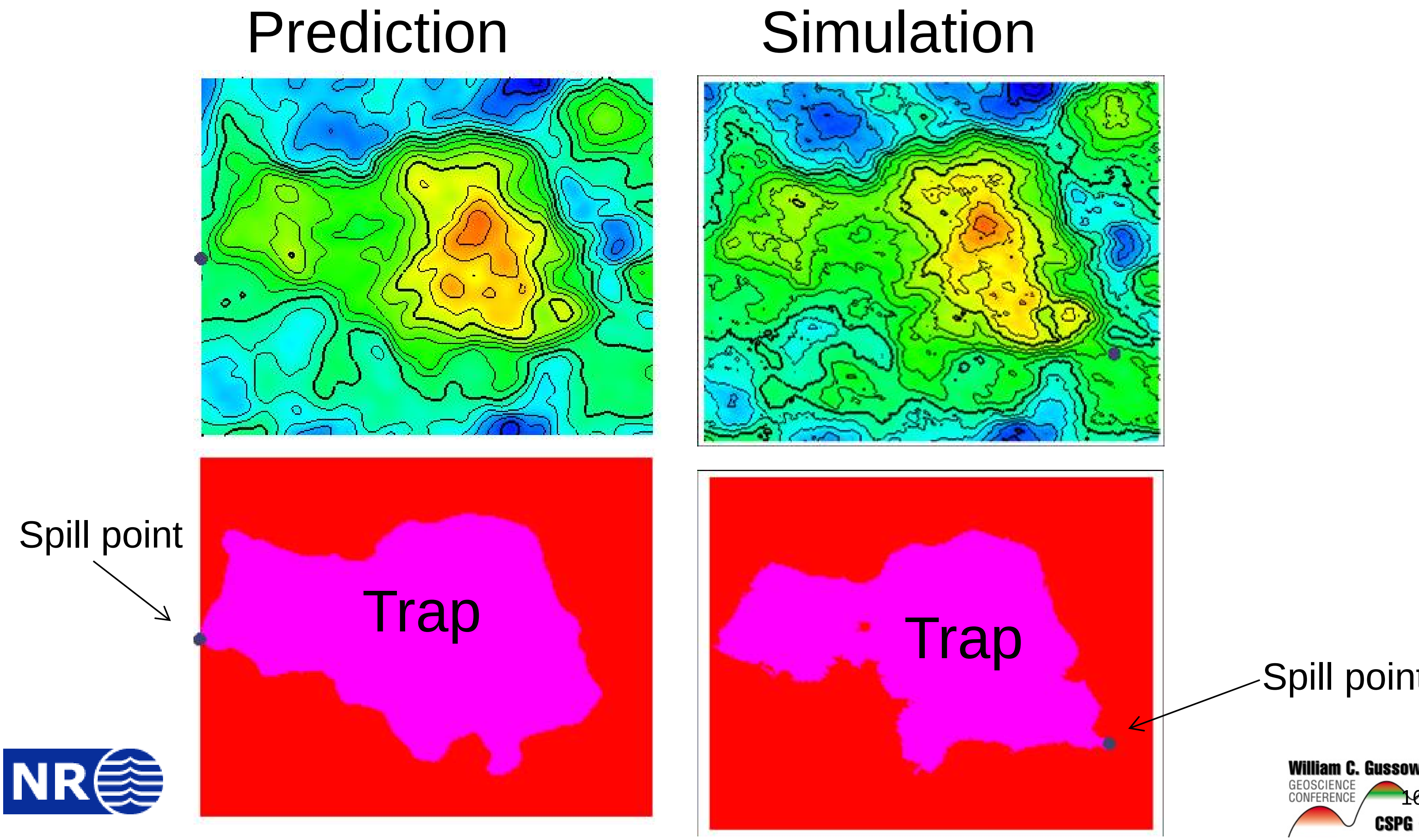
- Inefficient:
  - Monte Carlo convergence is slow: $\sim 1/\sqrt{N}$
  - High rejection rate

What is a spill point?

What is a spill point?

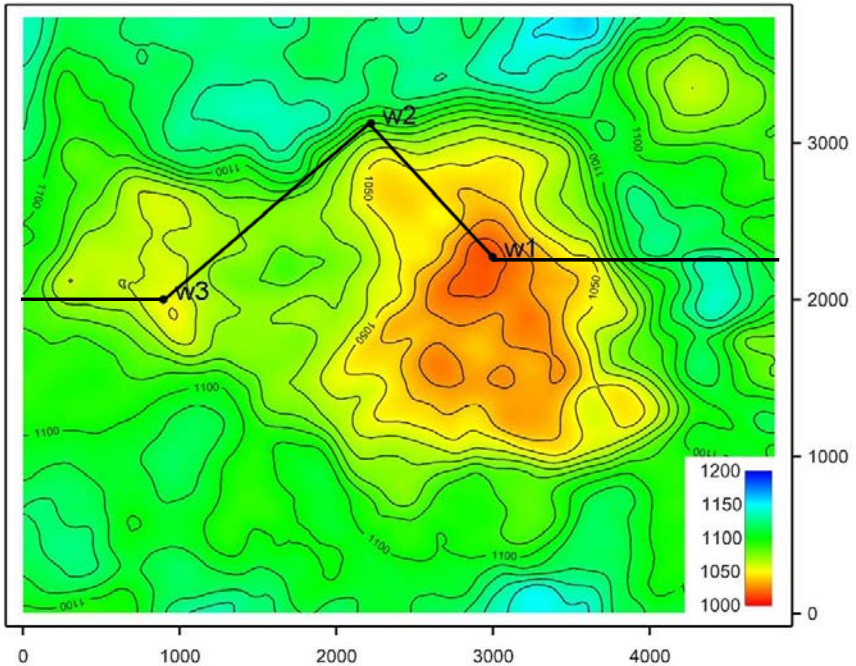


What is a spill point?

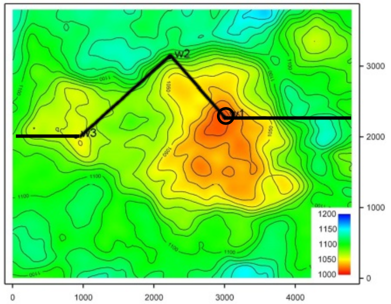


Conditioning on spill point

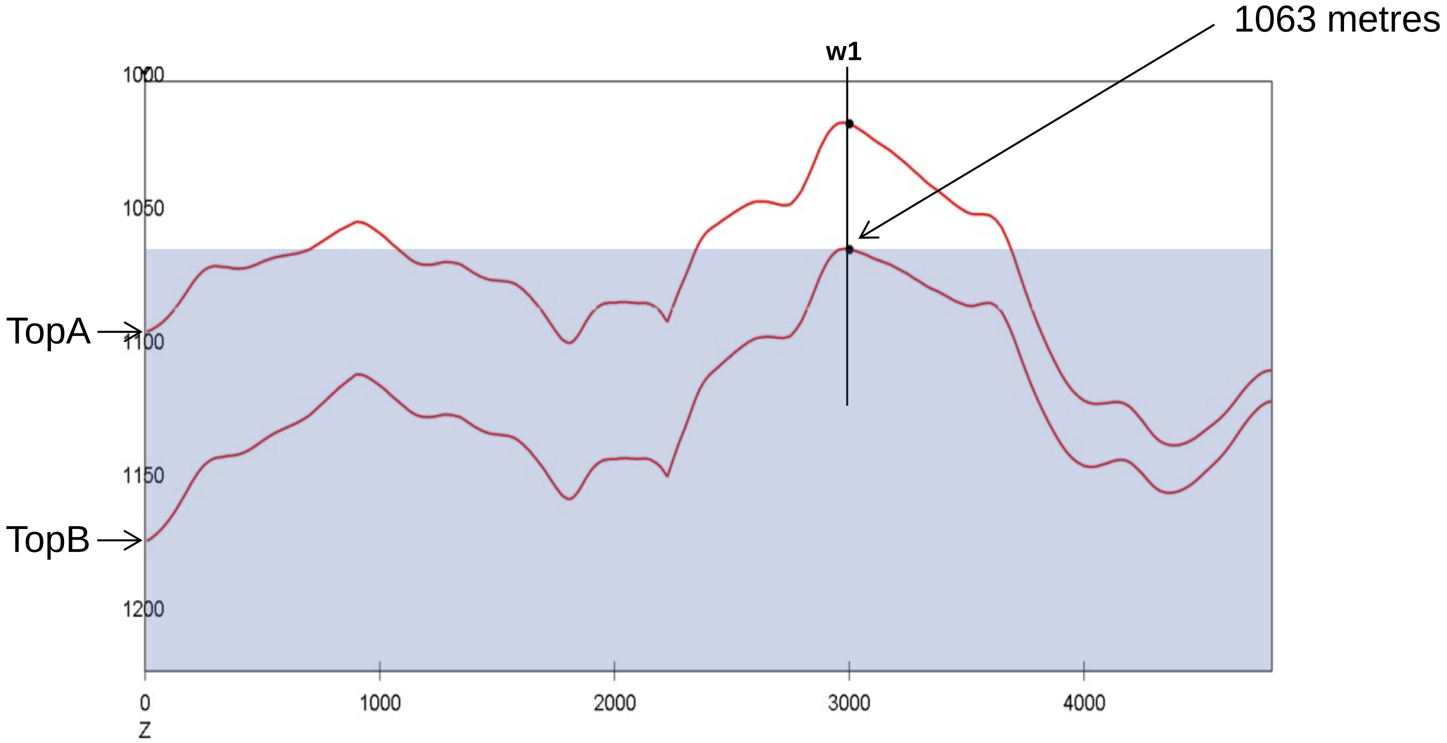
- Assume filled structure:
  OWC = spill point depth
- Reject if spill point is to deep or to shallow
- 1000 realizations
- Three cases
- Volume



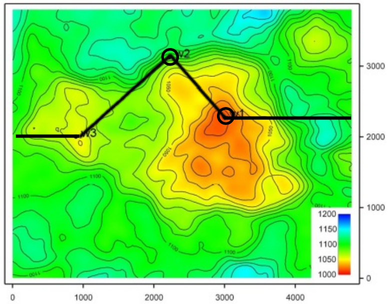
1 well



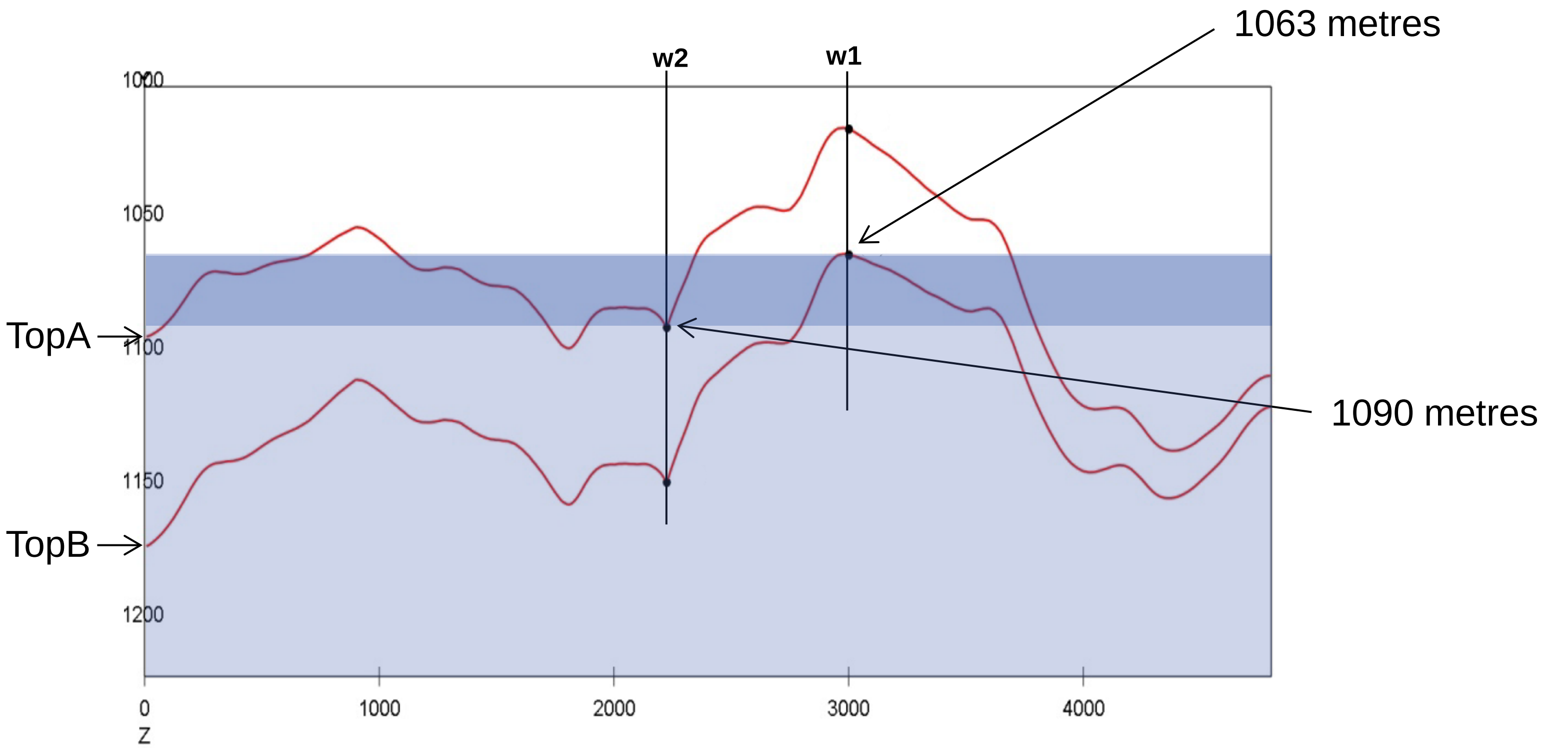
Spill point BELOW 1063 metres



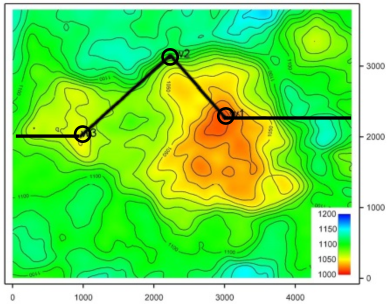
2 wells



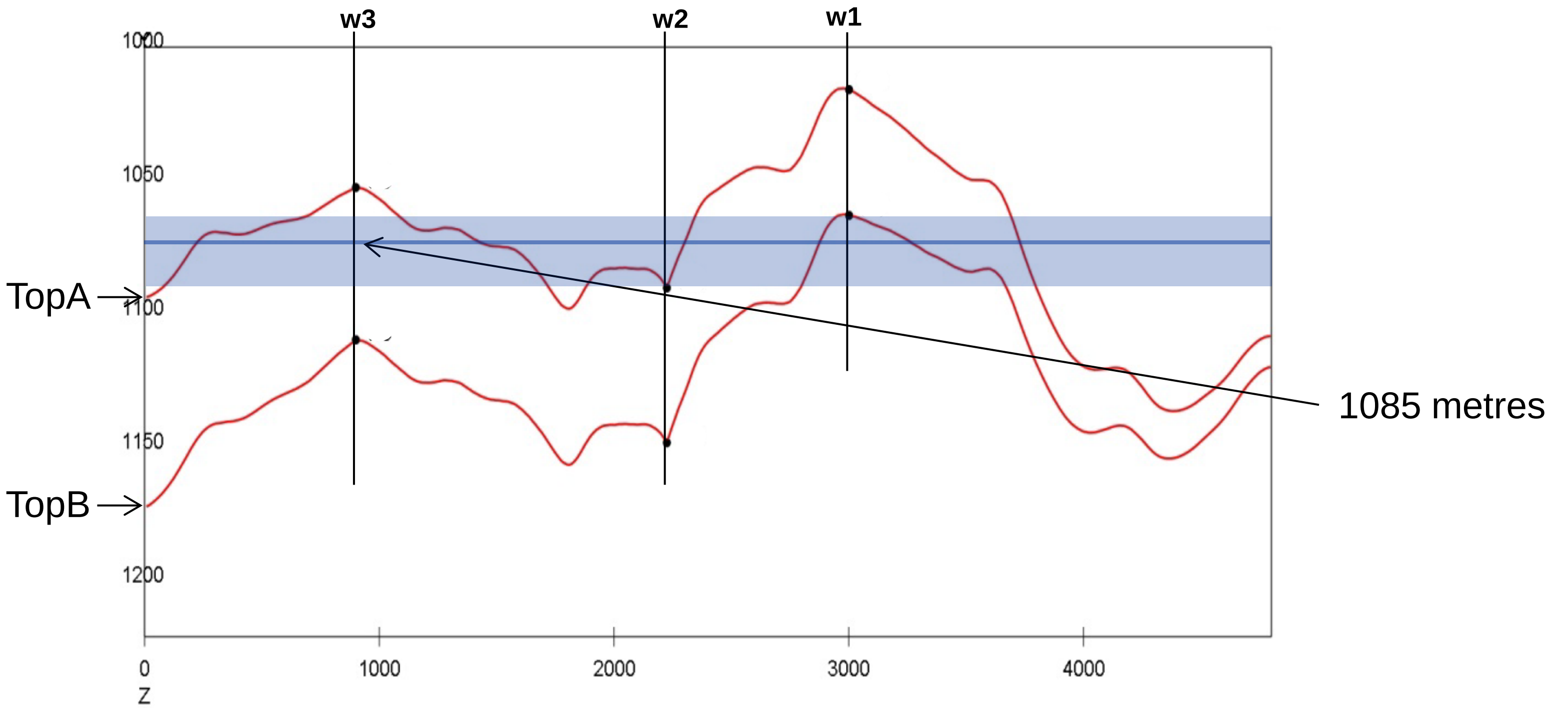
Spill point BELOW 1063 and ABOVE 1090 metres



3 wells

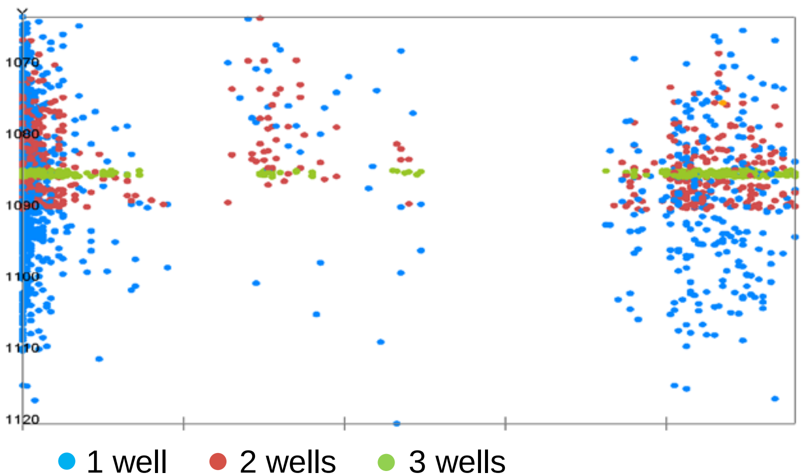


Spill point AT 1085 metre with a tolerance of ± 0.5 metres

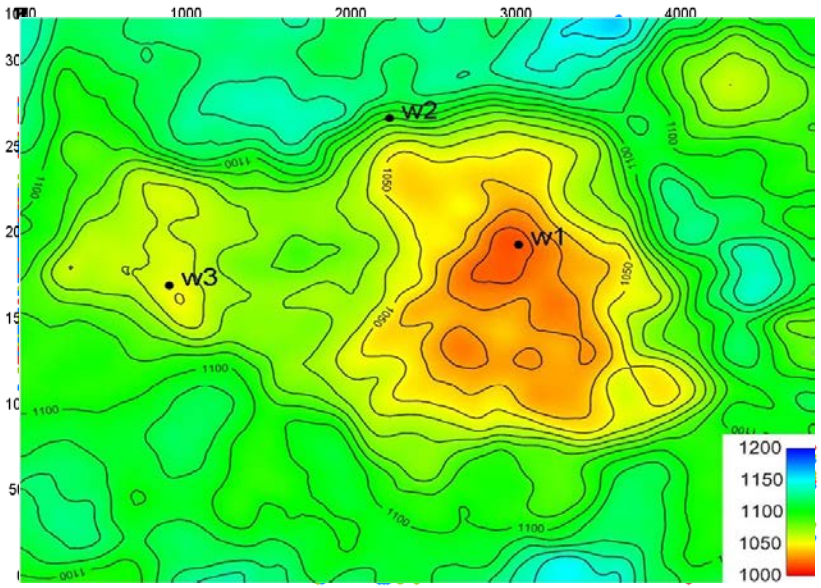


Spill point location

Vertical east-west projection

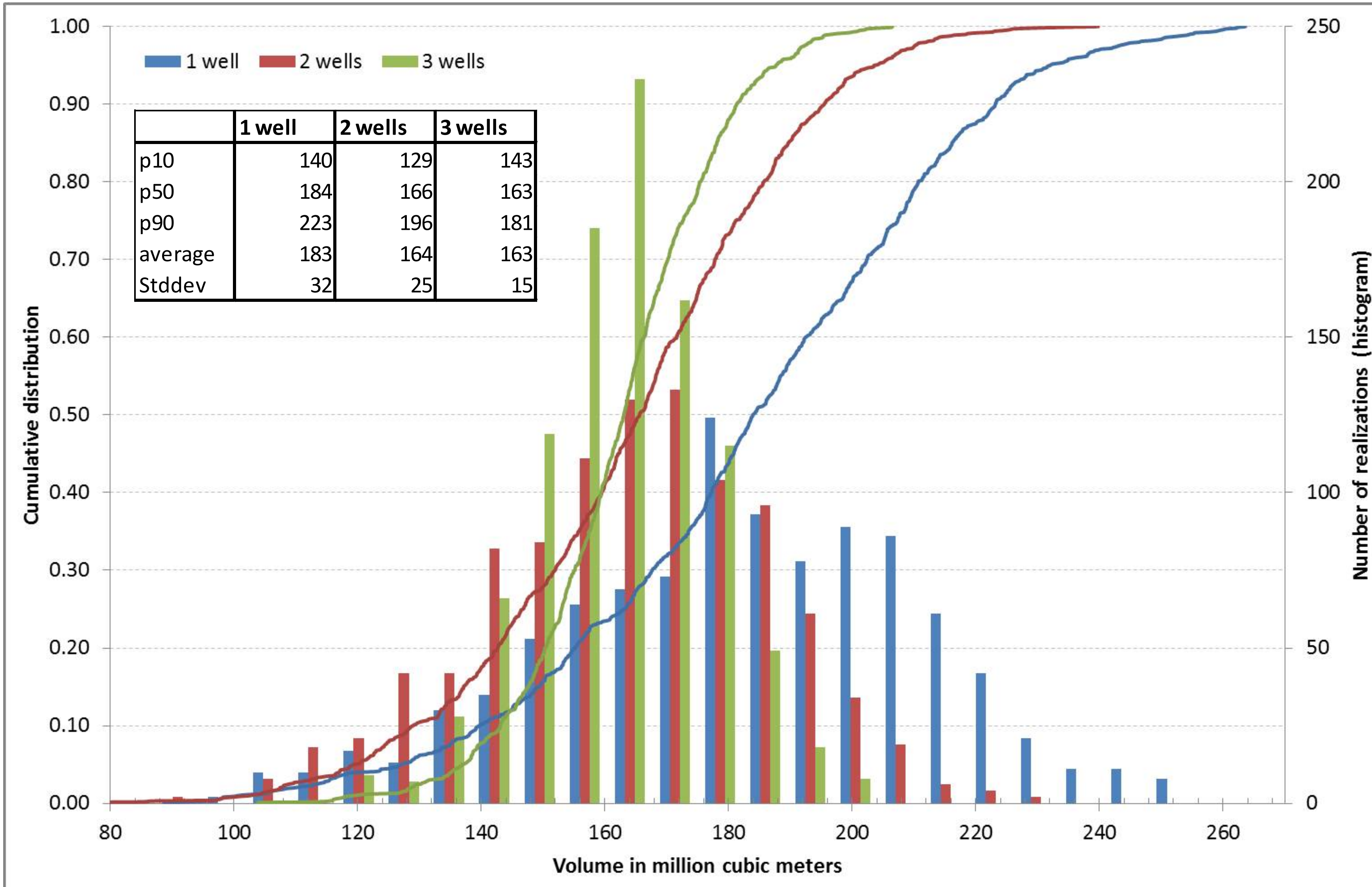


Map view



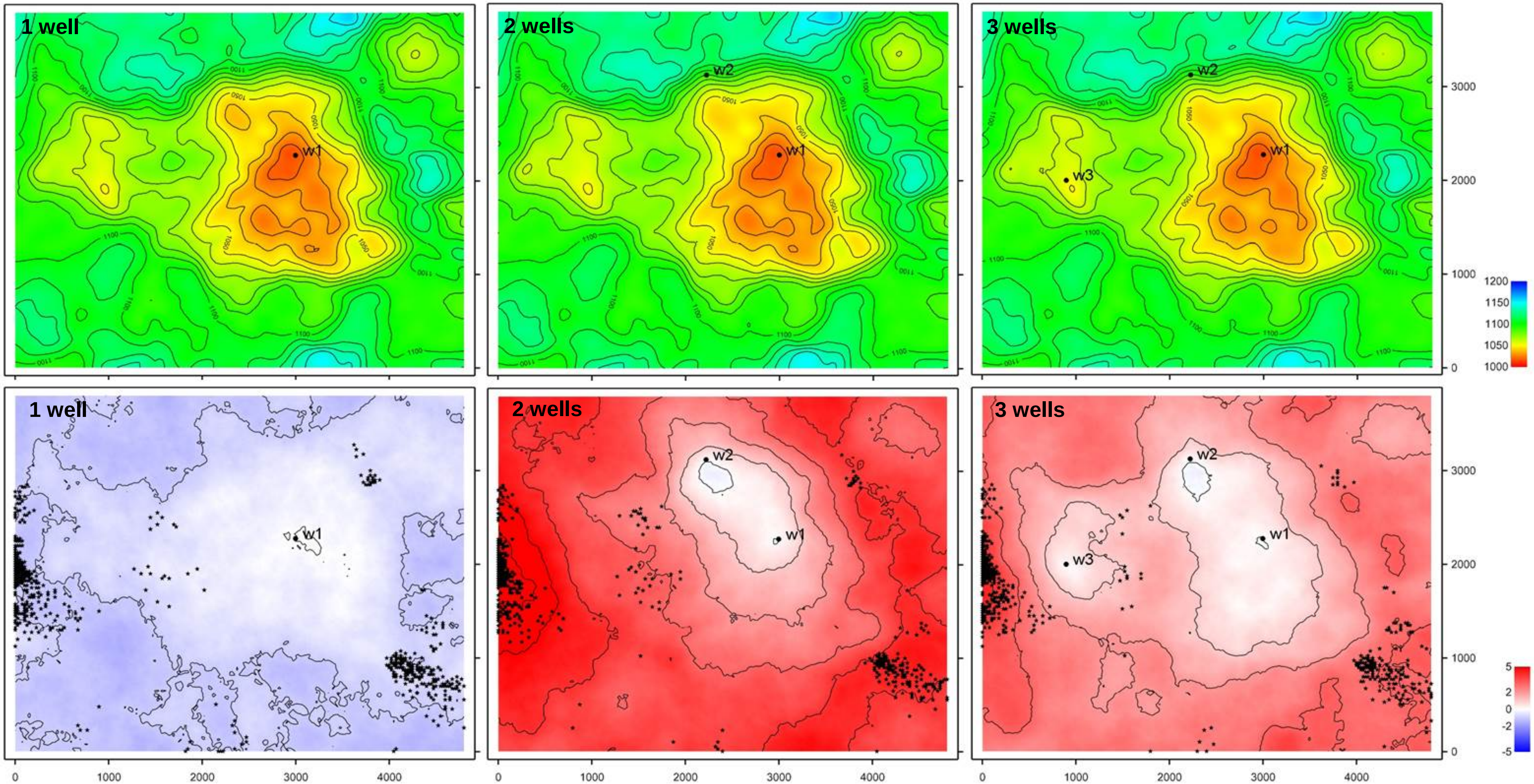
	Rejection rate	Average spill point depth
1 well	2 %	1086.7 ± 10.4
2 wells	54 %	1083.6 ± 4.8
3 wells	95 %	1085.0 ± 0.3

Volume statistics

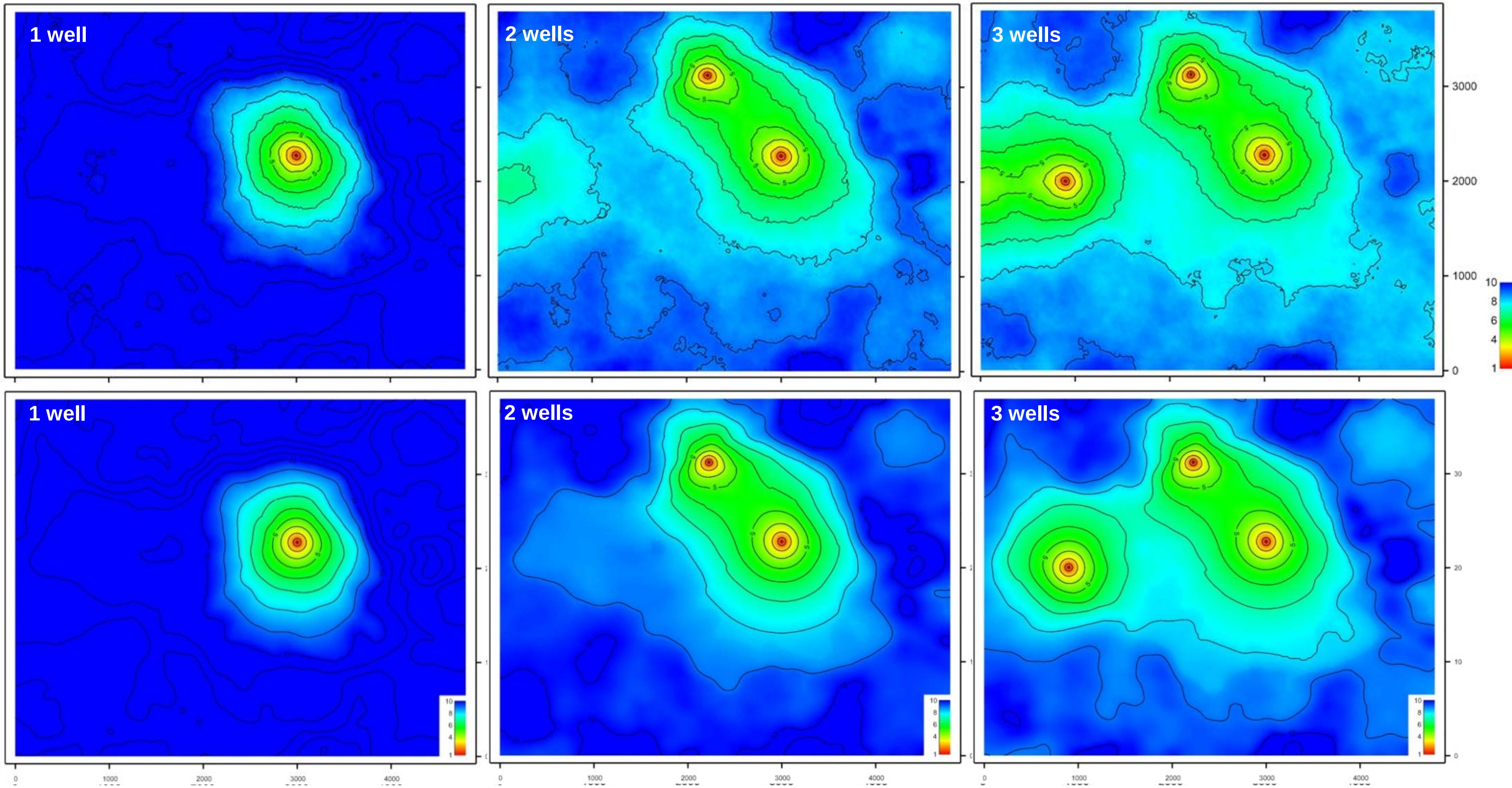


	1 well	2 wells	3 wells
p10	140	129	143
p50	184	166	163
p90	223	196	181
average	183	164	163
Stddev	32	25	15

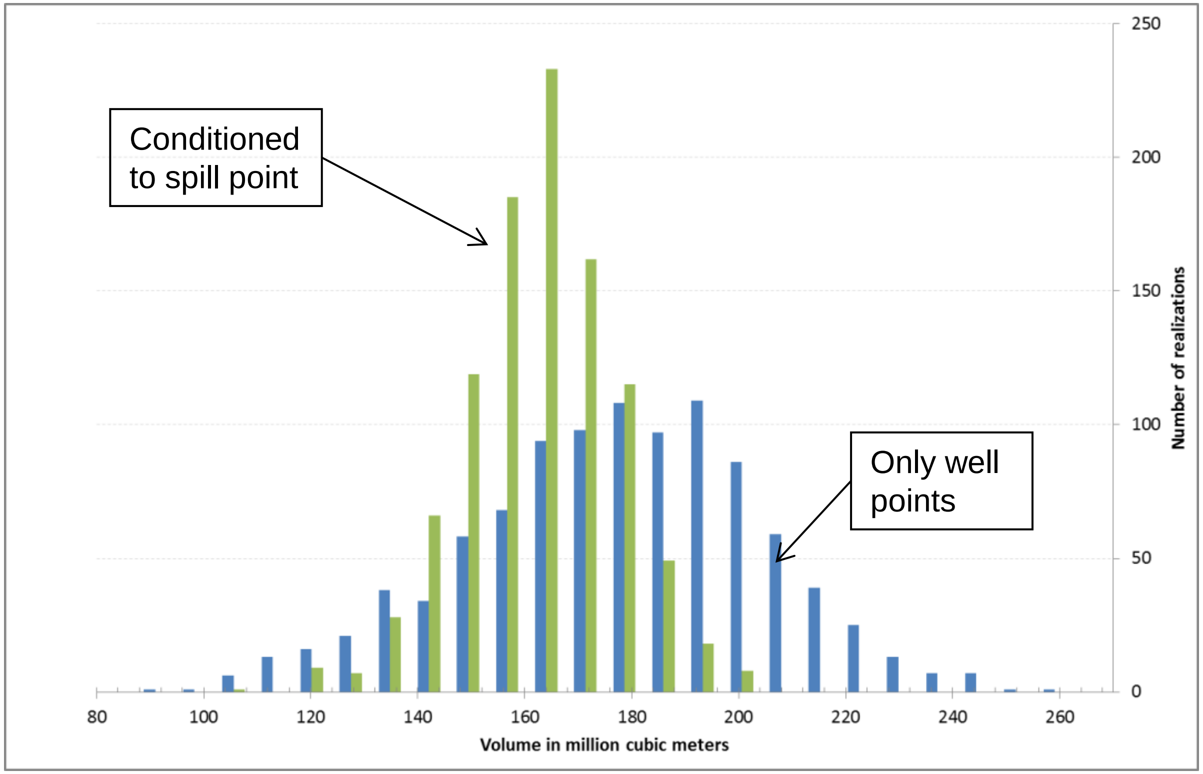
Average of 1000 realizations and deviation from prediction



Stddev of 1000 realizations and prediction error



Rejection sampling allows us to condition to spill point information



5 % acceptance rate

⇓

20 times slower

Prediction and simulation

Kriging predictor:

Estimated depth trend

Estimated trend at well data location

$$Z^*(\mathbf{x}) = \widehat{m}(\mathbf{x}) + \mathbf{k}(\mathbf{x})\mathbf{K}^{-1}(\mathbf{Z} - \widehat{\mathbf{m}})$$

Kriging vector

Kriging matrix

Well data

Simulation:

$$Z^s(\mathbf{x}) = m^s(\mathbf{x}) + \epsilon^s(\mathbf{x}) + \mathbf{k}(\mathbf{x})\mathbf{K}^{-1}(\mathbf{Z} - \underbrace{\mathbf{m}^s - \boldsymbol{\epsilon}^s})$$

Simulated depth trend

Simulated residual

Simulated trend and residual at well data location

$$\boxed{\mathrm{E}[\mathrm{Z}^s(\mathbf{x})] = \mathrm{Z}^*(\mathbf{x}) \quad \text{and} \quad \mathrm{Var}[\mathrm{Z}^s(\mathbf{x})]^{1/2} = \sigma^*(\mathbf{x})}$$

Fast simulation idea

Fast simulation (with trends):

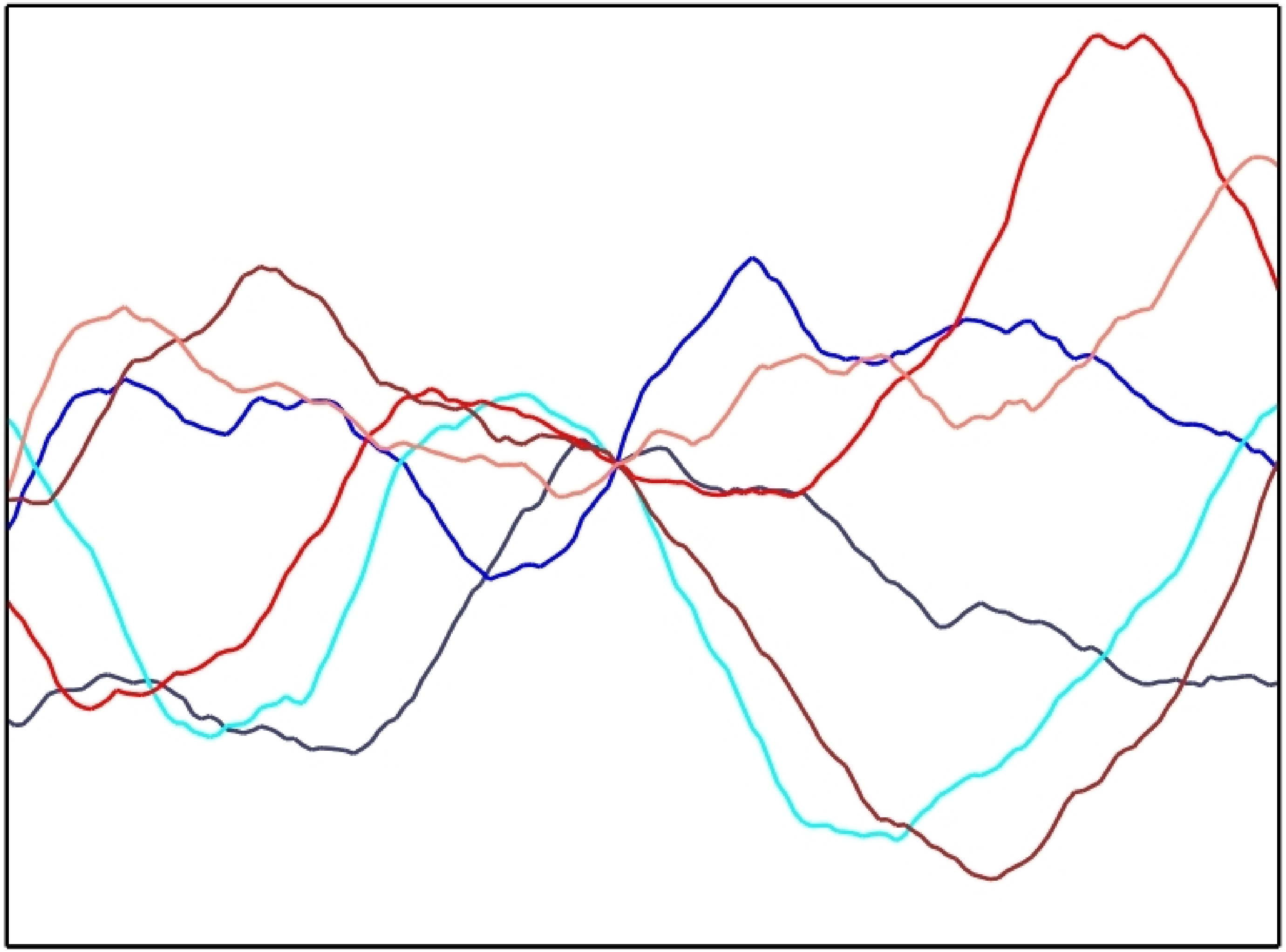
$$Z^s(\mathbf{x}) = Z^*(\mathbf{x}) + \left[m^s(\mathbf{x}) - \widehat{m}(\mathbf{x}) + \epsilon^s(\mathbf{x})\right] \frac{\sigma^*(\mathbf{x})}{\hat{\sigma}(\mathbf{x})}$$

Simulation:

$$Z^s(\mathbf{x}) = \underset{\text{Simulated depth trend}}{m^s(\mathbf{x})} + \underset{\text{Simulated residual}}{\epsilon^s(\mathbf{x})} + \mathbf{k}(\mathbf{x})\mathbf{K}^{-1}(\mathbf{Z} \underbrace{- \mathbf{m}^s - \boldsymbol{\epsilon}^s}_{\text{Simulated trend and residual at well data location}})$$

$$\boxed{\mathrm{E}[\mathrm{Z}^s(\mathbf{x})] = \mathrm{Z}^*(\mathbf{x}) \quad \text{and} \quad \mathrm{Var}[\mathrm{Z}^s(\mathbf{x})]^{1/2} = \sigma^*(\mathbf{x})}$$

6 samples, ordinary simulation



Closing remarks

- Rejection sampling handles non-linear constraints
  - Spill points (global constraint)
  - Horizontal well path (local constraints)
- Volume uncertainty reduced
- Efficiency can be improved
  - Fast approximate conditional simulation
  - Efficient algorithms
    - Transformations
    - Iterations
    - Approximations