Sequential optimization of oil production from multiple reservoirs under uncertainty

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Discrete time optimization under uncertainty

We consider the oil production from a field consisting of n reservoirs that share a processing facility with a constant process capacity K. The production from each reservoir is described as a discrete time process:

$$
\begin{aligned}
q_{ik} &= \text{The production from the } i\text{th reservoir in the } k\text{th period,} \\
Q_{ik} &= \text{The cum. production from the } i\text{th reservoir after the } k\text{th period} \\
&= \sum_{j=1}^{k} q_{ij}
\end{aligned}
$$

We also define $Q_{i0} = 0$.

Discrete time optimization under uncertainty

The maximum amount of oil that can be produced from the *i*th reservoir within the *k*th period given no other restrictions, is:

$$f_i(Q_{i,k-1}) = D_i(V_i - Q_{i,k-1}),$$

where $V_i > 0$ and $D_i \in [0, 1]$ are *random variables* and denote the *recoverable volume* and *decline rate* of the *i*th reservoir.

x_{ik} = quota assigned to the *i*th reservoir during the *k*th period

$\boldsymbol{x}_k = (x_{1k}, \ldots, x_{nk})$

The actual production volumes are then given by:

$$q_{ik} = q_{ik}(x_{ik}) = \min\{f_i(Q_{i,k-1}), x_{ik}\},$$

where the quotas are chosen so that $\sum_{i=1}^{n} x_{ik} = K$.

Discrete time optimization under uncertainty

If $x_{ik} \leq f_i(Q_{i,k-1})$, it follows that $q_{ik} = x_{ik}$. If this holds for all reservoirs, all quotas are fully utilized, and we get that:

$$\sum_{i=1}^{n} q_{ik} = \sum_{i=1}^{n} x_{ik} = K.$$

If $x_{jk} > f_j(Q_{j,k-1})$ for some j, the quota for this reservoir cannot be fully utilized, i.e., $q_{jk} < x_{jk}$. Hence, in this case:

$$\sum_{i=1}^{n} q_{ik} < \sum_{i=1}^{n} x_{ik} = K.$$

A good production strategy should aim at utilizing the quotas as much as possible for all reservoirs.

Short-term optimization under uncertainty

In order to formulate the optimization problem, we introduce:

$$Y_k = Y_k(\boldsymbol{x}_k) = \sum_{j=1}^{n} q_{jk}(x_{jk}) = \sum_{j=1}^{n} \min\{f_j(Q_{j,k-1}), x_{jk}\}, \quad k = 1, 2, \ldots.$$

Considering the kth time period, the objective is to choose $\boldsymbol{x}_k$ so that $E[Y_k(\boldsymbol{x}_k)]$ is maximized subject to the processing capacity constraint:

$$\sum_{i=1}^{n} x_{ik} = K.$$

Note that by using this approach at each step, the focus is on the upcoming time period only.

Short-term optimization under uncertainty

In order to solve the short-term optimization problem, we introduce the Lagrange function:

$$\Lambda_S(\boldsymbol{x}_k, \lambda) = \Phi_S(\boldsymbol{x}_k) - \lambda \Psi(\boldsymbol{x}_k),$$

where λ denotes the Lagrange multiplier, and where:

$$\Phi_S(\boldsymbol{x}_k) = E[Y_k(\boldsymbol{x}_k)],$$
$$\Psi(\boldsymbol{x}_k) = \sum_{i=1}^{n} x_{ik} - K.$$

A stationary point for the Lagrange function is then found by solving the equation:

$$\nabla \Phi_S(\boldsymbol{x}_k) = \lambda \nabla \Psi(\boldsymbol{x}_k),$$

subject to the restriction that $\Psi(\boldsymbol{x}_k) = 0$.

Short-term optimization under uncertainty

It is easy to verify that Φ_S is a *concave* function. Thus, the stationary point will be a maximum point. Moreover, for $i = 1, \ldots, n$, we get:

$$\begin{aligned}
\frac{\partial}{\partial x_{ik}} \Phi_S(\boldsymbol{x}_k) &= \frac{\partial}{\partial x_{ik}} E[Y_k(\boldsymbol{x}_k)] \\
&= E[\frac{\partial}{\partial x_{ik}} \sum_{j=1}^{n} \min\{f_j(Q_{j,k-1}), x_{jk}\}] \\
&= E[\frac{\partial}{\partial x_{ik}} \min\{f_i(Q_{i,k-1}), x_{ik}\}] \\
&= E[I(f_i(Q_{i,k-1}) > x_{ik})] \\
&= P(f_i(Q_{i,k-1}) > x_{ik}).
\end{aligned}$$

where $I(\cdot)$ denotes the indicator function.

Short-term optimization under uncertainty

Finally, we get:

$$\nabla\Psi(\boldsymbol{x}_k) = (1, \ldots, 1).$$

Combining all this, we get that the optimal solution must satisfy:

$$P(f_i(Q_{i,k-1}) > x_{ik}) = \lambda, \quad i = 1, \ldots n,$$

for some value of λ, as well as the processing capacity constraint:

$$\sum_{i=1}^{n} x_{ik} = K.$$

Note: The short-term strategy attempts to distribute the available processing capacity between the reservoirs, and hence balance the risk between these.

Long-term optimization under uncertainty

In the deterministic case the optimal strategy is to make sure that the reservoirs with lowest decline rates are produced first. As a result, the tail-production will be dominated by the reservoirs with the highest decline rates. As a result the remaining volumes will be produced as fast as possible.

In order to improve the results for the stochastic case we introduce a different approach where more focus is put on the tail-production. In particular, we aim at finding a strategy where the tail-production can be done as fast as possible.

Long-term optimization under uncertainty

One way of evaluating the tail-production is by calculating its *potential center of mass*. In order to define this concept, we start out by considering the ith reservoir, and assume that we have completed $k-1$ periods of production.

Assuming that the reservoir is produced at maximum speed in all the periods following the kth period, we get that:

$$q_{i,k+1} = D_i(V_i - Q_{i,k-1} - q_{ik}),$$

$$\begin{aligned} q_{i,k+2} &= D_i(V_i - Q_{i,k} - q_{i,k+1}) = D_i(V_i - Q_{i,k-1} - q_{ik} - q_{i,k+1}) \\ &= D_i((V_i - Q_{i,k-1} - q_{ik}) - D_i(V_i - Q_{i,k-1} - q_{ik})) \\ &= D_i(1 - D_i)(V_i - Q_{i,k-1} - q_{ik}), \\ &\vdots \\ q_{i,k+h} &= D_i(V_i - Q_{i,k+h-2} - q_{i,k+h-1}) = D_i(1 - D_i)^{h-1}(V_i - Q_{i,k-1} - q_{ik}). \end{aligned}$$

Long-term optimization under uncertainty

The *potential center of mass* for the tail-production after the kth period of the ith reservoir, expressed as a function of x_{ik}, and denoted by $Z_{ik}(x_{ik})$, can now be defined as:

$$
\begin{aligned}
Z_{ik}(x_{ik}) &= \sum_{h=1}^{\infty} h \cdot q_{i,k+h} \\
&= \sum_{h=1}^{\infty} h \cdot D_i(1 - D_i)^{h-1}(V_i - Q_{i,k-1} - q_{ik}(x_{ik})) \\
&= D_i(V_i - Q_{i,k-1} - q_{ik}(x_{ik})) \sum_{h=1}^{\infty} h \cdot (1 - D_i)^{h-1} \\
&= (V_i - Q_{i,k-1} - q_{ik}(x_{ik})) \cdot D_i^{-1}.
\end{aligned}
$$

Long-term optimization under uncertainty

The potential center of mass for the tail-production after the kth period of all reservoirs combined, is defined as:

$$Z_k(\boldsymbol{x}_k) = \sum_{j=1}^{n} Z_{jk}(x_{jk}) = \sum_{j=1}^{n} (V_j - Q_{j,k-1} - q_{jk}(x_{jk})) \cdot D_j^{-1}.$$

We seek a strategy where the expected potential center of mass is as low as possible, since this implies that the remaining volumes can be produced as fast as possible.

Long-term optimization under uncertainty

In order to solve the long-term optimization problem, we again introduce the Lagrange function:

$$\Lambda_L(\boldsymbol{x}_k, \lambda) = \Phi_L(\boldsymbol{x}_k) + \lambda\Psi(\boldsymbol{x}_k),$$

where λ in this case conveniently denotes the *negative* Lagrange multiplier, and where:

$$\Phi_L(\boldsymbol{x}_k) = E[Z_k(\boldsymbol{x}_k)],$$

$$\Psi(\boldsymbol{x}_k) = \sum_{i=1}^{n} x_{ik} - K.$$

A stationary point for the Lagrange function is then found by solving the equation:

$$-\nabla\Phi_L(\boldsymbol{x}_k) = \lambda\nabla\Psi(\boldsymbol{x}_k),$$

subject to the restriction that $\Psi(\boldsymbol{x}_k) = 0$.

Long-term optimization under uncertainty

It is easy to verify that Φ_L is a *convex* function. Thus, the stationary point will be a minimum point. Moreover, for $i = 1, \ldots, n$ we get:

$$\begin{aligned}
-\frac{\partial}{\partial x_{ik}} \Phi_L(\boldsymbol{x}_k) &= -\frac{\partial}{\partial x_{ik}} E[Z_k(\boldsymbol{x}_k)] \\
&= -E[\frac{\partial}{\partial x_{ik}} Z_k(\boldsymbol{x}_k)] \\
&= -E[\frac{\partial}{\partial x_{ik}} \sum_{j=1}^{n} (V_j - Q_{j,k-1} - q_{jk}(x_{jk})) \cdot D_j^{-1}] \\
&= E[\frac{\partial}{\partial x_{ik}} \min\{f_i(Q_{i,k-1}), x_{ik}\} \cdot D_i^{-1}] \\
&= E[I(f_i(Q_{i,k-1}) > x_{ik}) \cdot D_i^{-1}].
\end{aligned}$$

Long-term optimization under uncertainty

Finally, as in the previous case, we get:

$$\nabla\Psi(\boldsymbol{x}_k) = (1, \ldots, 1).$$

Combining all this, we get that the optimal solution must satisfy:

$$E[I(f_i(Q_{i,k-1}) > x_{ik}) \cdot D_i^{-1}] = \lambda, \quad i = 1, \ldots, n,$$

for some value of λ, as well as the processing capacity constraint:

$$\sum_{i=1}^{n} x_{ik} = K.$$

Note: The long-term strategy tends to give priority to reservoirs with smaller decline rates.

Handling the uncertainty

- By assessing distributions for the reservoir parameters, the expected values needed in the calculations of the optimal solutions can be computed.
- As the production develops, more information about the production parameters is gained. Hence, the uncertainty distributions must be updated (using Bayes' theorem).
- As a result of the updating, V_i and D_i typically become stochastically dependent even when they are independent apriori.
- The updated joint distributions of V_i and D_i can be simulated using a combination of rejection sampling and the *Metropolis-Hastings algorithm*.

A numerical example

We consider a simple numerical example, where $n = 2$, and where V_1 and V_2 are lognormally distributed apriori, while D_1 and D_2 are uniformly distributed apriori.

The reservoirs will be producing in 25 periods and processed on a facility with a capacity of $K = 1.2$ million barrels of oil per period.

i	$E[V_i]$	$SD[V_i]$	V_i	D_i^{min}	D_i^{max}	D_i
1	12.0	2.0	12.0	0.20	0.30	0.25
2	12.0	2.0	12.0	0.05	0.15	0.10

Table: Reservoir parameters.

Given the true values of the reservoir parameters, the optimal strategy is a *strict priority rule* where the reservoir with the lowest decline rate, i.e., Reservoir 2, is given top priority.

Results

	Total result	Disc. result
Short-term strategy	22.15	20.26
Long-term strategy	22.93	20.92
Deterministic strategy	22.94	20.94

Table: Results of the simulations for the three strategies.

Production profiles for the short-term strategy

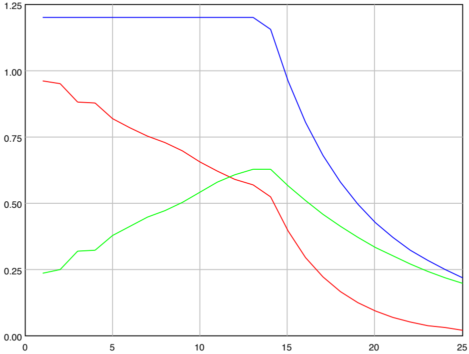

Figure: *Production profiles using short-term strategy for Reservoir 1 (red curve), Reservoir 2 (green curve), and Total production (blue curve)*

Production profiles for the long-term strategy

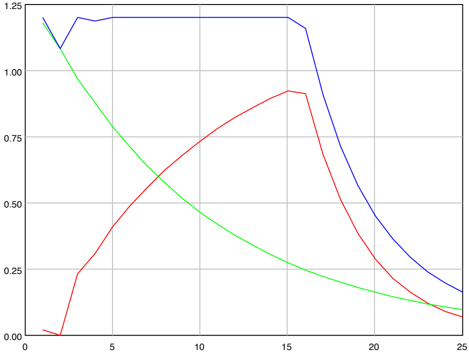

Figure: *Production profiles using long-term strategy for Reservoir 1 (red curve), Reservoir 2 (green curve), and Total production (blue curve)*

Production profiles for the deterministic strategy

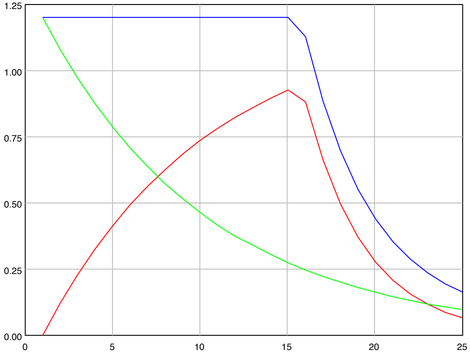

Figure: *Production profiles using the deterministic strategy for Reservoir 1 (red curve), Reservoir 2 (green curve), and Total production (blue curve)*

Conclusions

- A framework for optimizing oil production from several reservoirs sharing a common processing facility when the reservoir parameters are not known is proposed
- Both a short-term strategy and a long-term strategy have been analysed
- Both strategies are determined using step by step forward optimization making the calculations simple and efficient compared to full scale stochastic dynamic optimization
- Numerical studies have shown that the long-term strategy is performing better than the short-term strategy.
- Future work:
  - Multiphase production (i.e., oil/gas/water)
  - More complex production models