The Natvig measures of component importance in repairable systems applied to an offshore oil and gas production system

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In the present paper the Natvig measures of component importance for repairable systems, and its extended version are applied to an offshore oil and gas production system. According to the extended version of the Natvig measure a component is important if both by failing it strongly reduces the expected system uptime and by being repaired it strongly reduces the expected system downtime. The results include a study of how different distributions affect the ranking of the components. All numerical results are computed using discrete event simulation. In a companion paper (Huseby, Eide, Isaksen, Natvig, and Gåsemyr 2008) the advanced simulation methods needed in these calculations are decribed.

1 BASIC IDEAS, CONCEPTS AND RESULTS

Intuitively it seems that components that by failing strongly reduce the expected remaining system lifetime are very important. This is at least true during the system development phase. This is the motivation for the (Natvig 1979) measure of component importance in nonrepairable systems. In (Natvig 1982) a stochastic representation of this measure was obtained by considering the random variable:

$$Z_i = Y_i^1 - Y_i^0, \tag{1}$$

where:

$Y_i^0 =$ The remaining system lifetime just *after* the failure of the ith component.

$Y_i^1 =$ The remaining system lifetime just *after* the failure of the ith component, which, however, immediately undergoes a minimal repair; i.e., it is repaired to have the same distribution of remaining lifetime as it had just before failing.

Thus, Z_i can be interpreted as the increase in system lifetime due to a minimal repair of the ith component at failure. The Natvig measure of importance of the ith component is then defined as:

$$I_N^{(i)} = \frac{EZ_i}{\sum_{j=1}^n EZ_j}, \tag{2}$$

tacitly assuming $EZ_i < \infty$, $i = 1, \ldots, n$. Obviously

$$0 \leq I_N^{(i)} \leq 1, \qquad \sum_{i=1}^n I_N^{(i)} = 1. \tag{3}$$

For repairable systems we consider a time interval $[0, t]$ and start by introducing some basic random variables ($i = 1, \ldots, n$):

$T_{ij} =$ The time of the jth failure of the ith component, $j = 1, 2, \ldots,$

$S_{ij} =$ The time of the jth repair of the ith component, $j = 1, 2, \ldots$,

where we define $S_{i0} = 0$. Let ($i = 1, \ldots, n$ and $j = 1, 2 \ldots$):

$U_{ij} = T_{ij} - S_{ij-1} =$ The length of the jth lifetime of the ith component.

$D_{ij} = S_{ij} - T_{ij} =$ The length of the jth repair time of the ith component.

We assume that U_{ij} has an absolutely continuous distribution $F_i(t)$ with density $f_i(t)$ letting $\bar{F}_i(t) \stackrel{d}{=} 1 - F_i(t)$. Furthermore, D_{ij} is assumed to have an absolutely continuous distribution $G_i(t)$ with density $g_i(t)$ letting $\bar{G}_i(t) \stackrel{d}{=} 1 - G_i(t)$. $EU_{ij} = \mu_i$, $ED_{ij} = \nu_i$ and all lifetimes and repair times are assumed independent.

Parallel to the nonrepairable case we argue that components that by failing strongly reduce the expected system uptime should be considered as very important. In order to formalize this, we introduce ($i = 1, \ldots, n$ and $j = 1, 2, \ldots$):

$T'_{ij} =$ The fictive time of the jth failure of the ith component after a fictive minimal repair of the component at T_{ij}.

$Y^0_{ij} =$ System uptime in the interval $[\min(T_{ij}, t), \min(T'_{ij}, t)]$ assuming that the ith component is failed throughout this interval.

$Y^1_{ij} =$ System uptime in the interval $[\min(T_{ij}, t), \min(T'_{ij}, t)]$ assuming that the ith component is functioning throughout this interval as a result of the fictive minimal repair.

In order to arrive at a stochastic representation similar to the nonrepairable case, see (1), we introduce the following random variables ($i = 1, \ldots, n$):

$$Z_{ij} = Y^1_{ij} - Y^0_{ij}, \qquad j = 1, 2, \ldots. \tag{4}$$

Thus, Z_{ij} can be interpreted as the fictive increase in system uptime in the interval $[\min(T_{ij}, t), \min(T'_{ij}, t)]$ as a result of the ith component being functioning instead of failed in this interval. Note that since the minimal repair is fictive, we have chosen to calculate the effect of this repair over the entire interval $[\min(T_{ij}, t), \min(T'_{ij}, t)]$ even though this interval may extend beyond the time of the real repair, S_{ij}.

In order to summarize the effects of all the fictive minimal repairs, we have chosen to simply add up these contributions. Note that the fictive minimal repair periods, i.e., the intervals of the form $[\min(T_{ij}, t), \min(T'_{ij}, t)]$, may sometimes overlap. Thus, at a given point of time we may have contributions from more than one fictive minimal repair. This is efficiently dealt with by the simulation methods presented in (Huseby, Eide, Isaksen, Natvig, and Gåsemyr 2008). Taking the expectation, we get:

$$E\Big[\sum_{j=1}^{\infty} I(S_{ij-1} \leq t) Z_{ij}\Big] \stackrel{d}{=} EY_i(t), \tag{5}$$

where I denotes the indicator function. The time dependent Natvig measure of the importance of the ith component in the time interval $[0, t]$ in repairable systems can then be defined as:

$$I_N^{(i)}(t) = \frac{EY_i(t)}{\sum_{j=1}^{n} EY_j(t)}. \tag{6}$$

We now also take a dual term into account where components that by being repaired strongly reduce the expected system downtime are considered very important. Introduce ($i = 1, \ldots, n$ and $j = 1, 2, \ldots$):

$S'_{ij} =$ The fictive time of the jth repair of the ith component after a fictive minimal failure of the component at S_{ij}.

$X^0_{ij} =$ System downtime in the interval $[\min(S_{ij}, t), \min(S'_{ij}, t)]$ assuming that the ith component is functioning throughout this interval.

$X^1_{ij} =$ System downtime in the interval $[\min(S_{ij}, t), \min(S'_{ij}, t)]$ assuming that the ith component is failed throughout this interval as a result of the fictive minimal failure.

We then introduce the following random variables parallel to (4) ($i = 1, \ldots, n$):

$$W_{ij} = X^1_{ij} - X^0_{ij}, \qquad j = 1, 2, \ldots. \tag{7}$$

In this case W_{ij} can be interpreted as the fictive increase in system downtime in the interval $[\min(S_{ij}, t), \min(S'_{ij}, t)]$ as a result of the ith component being failed instead of functioning in this interval.

Now adding up the contributions from the repairs at S_{ij}, $j = 1, 2, \ldots$, and taking the expectation, we get:

$$E\Big[\sum_{j=1}^{\infty} I(T_{ij} \leq t) W_{ij}\Big] \stackrel{d}{=} EX_i(t). \tag{8}$$

The time dependent dual Natvig measure of the importance of the ith component in the time interval $[0, t]$ in repairable systems can then be defined as:

$$I_{N,D}^{(i)}(t) = \frac{EX_i(t)}{\sum_{j=1}^{n} EX_j(t)}. \tag{9}$$

An extended version of (6) is given by:

$$\bar{I}_N^{(i)}(t) = \frac{EY_i(t) + EX_i(t)}{\sum_{j=1}^{n}[EY_j(t) + EX_j(t)]}. \quad (10)$$

In (Natvig 1985) it is shown that:

$$P(T'_{ij} - S_{ij-1} > t) \quad (11)$$

$$= \bar{F}_i(t) + \int_0^t f_i(t-u)\frac{\bar{F}_i(t)}{\bar{F}_i(t-u)}du$$

$$= \bar{F}_i(t)[1 - \ln \bar{F}_i(t)].$$

Hence, applying (11) we get:

$$\int_0^\infty \bar{F}_i(t)(-\ln \bar{F}_i(t))dt \quad (12)$$

$$= \int_0^\infty \bar{F}_i(t)[1 - \ln \bar{F}_i(t)]dt - \int_0^\infty \bar{F}_i(t)dt$$

$$= E(T'_{ij} - S_{ij-1}) - E(T_{ij} - S_{ij-1})$$

$$= E(T'_{ij} - T_{ij}) \stackrel{d}{=} \mu_i^p.$$

Accordingly, this integral equals the expected prolonged lifetime of the ith component due to a minimal repair. Completely parallel we have:

$$\int_0^\infty \bar{G}_i(t)(-\ln \bar{G}_i(t))dt = E(S'_{ij} - S_{ij}) \stackrel{d}{=} \nu_i^p. \quad (13)$$

Let $A_i(t)$ be the availability of the ith component at time t, i.e., the probability that the component is functioning at time t. The corresponding stationary availabilities are given by:

$$A_i = \lim_{t\to\infty} A_i(t) = \frac{\mu_i}{\mu_i + \nu_i}, \qquad i = 1, \dots, n. \quad (14)$$

Introduce $\boldsymbol{A}(t) = (A_1(t), \dots, A_n(t))$ and $\boldsymbol{A} = (A_1, \dots, A_n)$. Now the availability of the system at time t is given by $h(\boldsymbol{A}(t))$, where h is the system's reliability function.

The (Birnbaum 1969) measure at time t is given by:

$$I_B^{(i)}(t) = h(1_i, \boldsymbol{A}(t)) - h(0_i, \boldsymbol{A}(t)), \quad (15)$$

which is the probability that the ith component is critical for system functioning at time t. The corresponding stationary measure is given by:

$$I_B^{(i)} = \lim_{t\to\infty} I_B^{(i)}(t) = h(1_i, \boldsymbol{A}) - h(0_i, \boldsymbol{A}). \quad (16)$$

In (Natvig and Gåsemyr 2008) the following stationary versions of (6) and (10) are arrived at:

$$I_N^{(i)} = \lim_{t\to\infty} I_N^{(i)}(t) = \frac{[I_B^{(i)}/(\mu_i + \nu_i)]\mu_i^p}{\sum_{j=1}^{n}[I_B^{(j)}/(\mu_j + \nu_j)]\mu_j^p}. \quad (17)$$

$$\bar{I}_N^{(i)} = \lim_{t\to\infty} \bar{I}_N^{(i)}(t) \quad (18)$$

$$= \frac{[I_B^{(i)}/(\mu_i + \nu_i)](\mu_i^p + \nu_i^p)}{\sum_{j=1}^{n}[I_B^{(j)}/(\mu_j + \nu_j)](\mu_j^p + \nu_j^p)}.$$

Now consider the special case where the lifetime and repair time distributions are Weibull distributed; i.e.,

$$\bar{F}_i(t) = e^{-(\lambda_i t)^{\alpha_i}}, \qquad \lambda_i > 0,\, \alpha_i > 0,$$

$$\bar{G}_i(t) = e^{-(\gamma_i t)^{\beta_i}}, \qquad \gamma_i > 0,\, \beta_i > 0.$$

We then have:

$$\int_0^\infty \bar{F}_i(t)(-\ln \bar{F}_i(t))dt$$

$$= \frac{1}{\alpha_i}\frac{1}{\lambda_i}\int_0^\infty u^{1/\alpha_i + 1 - 1}e^{-u}du$$

$$= \frac{1}{\alpha_i}\frac{1}{\lambda_i}\Gamma(\frac{1}{\alpha_i} + 1) = \frac{\mu_i}{\alpha_i}.$$

Hence, (18) simplifies to:

$$\bar{I}_N^{(i)} = \frac{[I_B^{(i)}/(\mu_i + \nu_i)](\mu_i/\alpha_i + \nu_i/\beta_i)}{\sum_{j=1}^{n}[I_B^{(j)}/(\mu_j + \nu_j)](\mu_j/\alpha_j + \nu_j/\beta_j)}. \quad (19)$$

Now assume that α_i is increasing and λ_i changing in such a way that μ_i is constant. Hence, according to (14) the availability A_i is unchanged. Then $\bar{I}_N^{(i)}$ is decreasing in α_i. This is natural since a large $\alpha_i > 1$ corresponds to a strongly increasing failure rate and the effect of a minimal repair is small. Hence, according to $\bar{I}_N^{(i)}$ the ith component is of less importance. If on the other hand $\alpha_i < 1$ is small, we have a strongly decreasing failure rate and the effect of a minimal repair is large. Hence, according to $\bar{I}_N^{(i)}$ the ith component is of higher importance. A completely parallel argument is valid for β_i.

In the present paper the Natvig measures of component importance for repairable systems, given by (6), (9) and (10) are applied to an offshore oil and gas production system. In a companion paper (Huseby, Eide, Isaksen, Natvig, and Gåsemyr 2008) the advanced simulation methods needed in the calculations are decribed. In (Natvig and Gåsemyr 2008) a more thorough theoretical presentation of the Natvig measures for repairable systems and their stationary versions is given.

2 DESCRIPTION OF THE SYSTEM

We will now look at a West-African production site for oil and gas based on a memo (Signoret and Clave 2007). For this real life example we need to do some simplifications. Originally this is a multi-state system, which means that it has several functioning levels. In this paper, however, we are only considering binary systems. Thus a simplified definition of the system will be used. There are several different possible definitions, but we will use the following:

The oil and gas production site is said to be functioning if it can produce some amount of both oil and gas. Otherwise the system is failed.

Oil and gas are pumped up from one production well along with water. These substances are separated in a separation unit. We will assume this unit to function perfectly.

After being separated the oil is run through an oil treatment unit, which is also assumed to function perfectly. Then the treated oil is exported through a pumping unit.

The gas is sent through two compressors which compress the gas. When both compressors are functioning, we get the maximum amount of gas. However, to obtain at least *some* gas production, it is sufficient that at least one of the compressors is functioning. If this is the case, the uncompressed gas is burned in a flare, which is assumed to function perfectly. The compressed gas is run through a unit where it is dehydrated. This is called a TEG (Tri-Ethylene Glycol) unit. After being dehydrated, the gas is ready to be exported. Some of the gas is used as fuel for the compressors.

The water is first run through a water treatment unit. This unit cleanses the water so that it legally can be pumped back into the wells to maintain the pressure, or back into the sea. If the water treatment unit fails, the whole production stops.

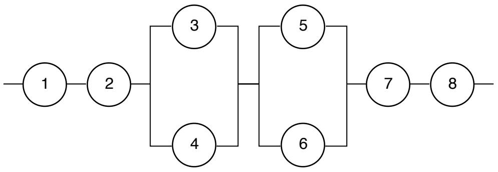

Figure 1: Model of oil and gas production site.

The components in the system also need electricity which comes from two generators. At least one generator must function in order to produce some oil and gas. If both generators are failed, the whole system is failed. The generators are powered by compressed and dehydrated gas.

Thus, the simplified production site considered in the present paper, consists of the following 8 relevant components, which are assumed to operate independently:

1. Well: A production well where the oil and gas come from.
2. Water cleanser: A component which cleanses the water which is pumped up from the production well along with the oil and gas.
3. Generator 1: Generator providing electricity to the system.
4. Generator 2: The same as Generator 1.
5. Compressor 1: A compressor which compresses the gas.
6. Compressor 2: The same as Compressor 1.
7. TEG: A component where the gas is dehydrated.
8. Oil export pump: An oil export pump.

The structure of the system is shown in Figure 1. The components 1, 2, 7 and 8 are all in series with the rest of the system, while the two generators, 3 and 4, operate in parallel with each other. Similarly the two compressors, 5 and 6, operate in parallel with each other.

Table 1: Failure rates, mean repair times and mean lifetimes of the components in the oil and gas production site

Component	Failure rate	ν_i	μ_i
1	$2.736 \cdot 10^{-4}$	7.000	3654.97
2	$8.208 \cdot 10^{-3}$	0.167	121.83
3 & 4	$1.776 \cdot 10^{-2}$	1.167	56.31
5 & 6	$1.882 \cdot 10^{-2}$	1.083	53.11
7	$1.368 \cdot 10^{-3}$	0.125	730.99
8	$5.496 \cdot 10^{-4}$	0.125	1819.51

Table 1 shows the given failure rates, mean repair times and mean lifetimes of the components in the system. The time unit is *days*. The mean lifetimes are considerably larger than the mean repair times. For some components (the well, the TEG unit and the oil export pump) the mean lifetimes are actually several years.

3 EXPONENTIALLY DISTRIBUTED LIFE- AND REPAIR TIMES

In this section we assume that the components have exponentially distributed life- and repair times. The failure rates in the lifetime distributions are the inverses of the mean lifetimes, while the repair rates are the inverses of the mean repair times. Thus, all the parameters needed in the simulations can be derived from Table 1. The time horizon t is set to 100000 days.

In Table 2 we see that $I_N^{(i)}(t)$ is equal to its extended version $\bar{I}_N^{(i)}(t)$. This is because $E[Y_i(t)]$ is very large compared to $E[X_i(t)]$ for all components. Hence, the contributions of the latter terms in (10) are too small to make any difference.

The reason for this is that the repair times of the components are much shorter than the corresponding lifetimes. Hence, the fictive prolonged repair times of the components due to the fictive minimal failures are much shorter than the fictive prolonged lifetimes due the fictive minimal repairs. Especially, the fictive prolonged repair times will, due to the much longer lifetimes, mostly end long before the next real repair. Hence, it is very unlikely that the fictive minimal failure periods will overlap. As a conclusion it is very sensible for this case study that $I_N^{(i)}(t)$ is equal to $\bar{I}_N^{(i)}(t)$.

We also observe from Table 2 that for the two equal measures the components 1, 2, 7 and 8 that are in series with the rest of the system have approximately the same importance. This can be seen by the following argument. Since $t = 100000$ days we have reached stationarity. Furthermore, for the exponential lifetime distribution $\mu_i^p = \mu_i$. If components i and j both are in series with the rest of the system, by conditioning on the state of component j and applying (14), the numerator of (17) equals $h(1_i, 1_j, \boldsymbol{A})A_iA_j$. By a parallel argument this is also the numerator of $I_N^{(j)}$.

Note also that the remaining components that are parts of parallel modules are much less important than the ones in series with the rest of the system. This is due to the very small unavailability $(1 - A_i)$ that appears as a common factor when factoring the numerator of (17). Indeed, in the exponential case we have, if $i = 3$ or 4 and $j = 5$ or 6, or vice versa, that this numerator equals

$$A_1A_2A_7A_8(1 - (1 - A_j)^2)(1 - A_i)A_i,$$

where all factors except $(1 - A_i)$ are close to 1. Furthermore, from Table 1 we see that all components 3, 4, 5 and 6 have almost identical unavailabilities, explaining why these components have identical importances.

The ranks of the component importance for the three versions of the Natvig measure are given in Table 3. We suggest to apply the common ranking based on the measures $I_N^{(i)}(t)$ and $\bar{I}_N^{(i)}(t)$.

4 GAMMA DISTRIBUTED LIFE- AND REPAIR TIMES

In this section we assume instead that the components have gamma distributed life- and repair times. More specifically, we assume that for $i = 1, \ldots, 8$, the lifetimes of the ith component have the densities:

$$f_i(t) = \frac{1}{(\beta_i)^{\alpha_i}\Gamma(\alpha_i)} t^{\alpha_i - 1} \exp(-t/\beta_i),$$

Table 2: Component importance using exponential distributions.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.244	0.371	0.244
2	0.249	0.267	0.249
3 & 4	0.005	0.080	0.005
5 & 6	0.005	0.077	0.005
7	0.247	0.033	0.246
8	0.241	0.013	0.241

Table 3: The ranks of the component importance for the three versions of the Natvig measure according to the results given in Table 2.

Measure	Rank
$I_N^{(i)}(t)$	$2 > 7 > 1 > 8 > 3 \approx 4 \approx 5 \approx 6$
$I_{N,D}^{(i)}(t)$	$1 > 2 > 3 \approx 4 > 5 \approx 6 > 7 > 8$
$\bar{I}_N^{(i)}(t)$	$2 > 7 > 1 > 8 > 3 \approx 4 \approx 5 \approx 6$

while the repair times of the ith component have the densities:

$$g_i(t) = \frac{1}{(\beta_i')^{\alpha_i'}\Gamma(\alpha_i')} t^{\alpha_i' - 1} \exp(-t/\beta_i').$$

Thus, for $i = 1, \ldots, 8$ and $j = 1, 2, \ldots$, we have:

$$E[U_{ij}] = \mu_i = \alpha_i\beta_i,$$

$$Var[U_{ij}] = \alpha_i(\beta_i)^2,$$

$$E[D_{ij}] = \nu_i = \alpha_i'\beta_i',$$

$$Var[D_{ij}] = \alpha_i'(\beta_i')^2,$$

where $\mu_1, \ldots, \mu_8$ and $\nu_1, \ldots, \nu_8$ are given in Table 1.

By choosing different values for the density parameters it is possible to alter the variances in the lifetime distributions and still keep the expectations fixed. In order to see the effect of this on the importance measures, we focus on component 1 where we consider five different parameter combinations for the lifetime distribution. For all these combinations, the expected lifetime is 3654.97 days, but the variance varies between $1.827 \cdot 10^3$ and $1.170 \cdot 10^6$. Table 4 lists these parameter combinations. For the remaining gamma densities we use the parameters listed in Table 5 and Table 6. All parameters are chosen such that the expectations in the life- and repair time distributions match the corresponding values given in Table 1. We also use the same time horizon $t = 100000$ days as in the previous section.

Tables 7, 8, 9, 10 and 11 display the results obtained from simulations using the parameters listed in

Table 4: Parameter sets for the lifetime distribution of component 1.

Set	α_1	β_1	Variance
1	7309.940	0.500	$1.827 \cdot 10^3$
2	550.033	6.645	$2.429 \cdot 10^4$
3	101.493	36.012	$1.316 \cdot 10^5$
4	45.687	80.000	$2.924 \cdot 10^5$
5	11.422	319.994	$1.170 \cdot 10^6$

Table 5: Parameters in the lifetime distributions of components $2, \ldots, 8$.

Component	α_i	β_i	Variance
2	30.000	4.062	$4.950 \cdot 10^2$
3 & 4	30.000	1.877	$1.057 \cdot 10^2$
5 & 6	10.000	5.311	$2.821 \cdot 10^2$
7	179.958	4.062	$2.969 \cdot 10^3$
8	218.219	8.338	$1.517 \cdot 10^4$

Table 6: Parameters in the repair time distributions of components $1, \ldots, 8$.

Component	α_i'	β_i'	Variance
1	3.500	2.000	$1.400 \cdot 10^1$
2	0.668	0.250	$4.175 \cdot 10^{-2}$
3 & 4	3.000	0.389	$4.540 \cdot 10^{-1}$
5 & 6	1.500	0.722	$7.819 \cdot 10^{-1}$
7	1.000	0.125	$1.563 \cdot 10^{-2}$
8	1.000	0.125	$1.563 \cdot 10^{-2}$

Tables 4, 5 and 6. As for the case with exponentially distributed life- and repair times, $I_N^{(i)}$ is equal to its extended version $\bar{I}_N^{(i)}$.

We now observe that for these two equal measures the components 1, 2, 7 and 8 that are in series with the rest of the system have different importances as opposed to the case with exponentially distributed life- and repair times. However, the remaining components that are parts of parallel modules are still much less important.

Furthermore, we see that the extended component importance of component 1 is increasing with increasing variances, and decreasing shape parameters α_1, all greater than 1, in its lifetime distribution. Since we have reached stationarity, this observation is in accordance with the discussion following (19) concerning the Weibull distribution.

Table 12 displays the ranks of the components according to the extended measure. Along with the increased importance, according to the extended measure, of component 1 as α_1 decreases, we observe from this table a corresponding improvement in its rank. All the other components are ranked in the same order for every value of α_1. This is as expected from (18) since the ordering is determined by its numerator. For all components except component 1 the numerator depends on the life- and repair time distributions of this component only through A_1, which is kept fixed when varying α_1. We also see that the components that are in series with the rest of the system are ranked according to the shape parameter α_i, such that components with smaller shape parameters are more important.

Table 7: Component importance using gamma distributions. Variance of component 1 lifetimes: $1.827 \cdot 10^3$.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.031	0.246	0.034
2	0.521	0.419	0.520
3 & 4	0.010	0.059	0.011
5 & 6	0.018	0.081	0.019
7	0.202	0.043	0.200
8	0.188	0.017	0.186

Table 8: Component importance using gamma distributions. Variance of component 1 lifetimes: $2.429 \cdot 10^4$.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.107	0.244	0.109
2	0.477	0.415	0.476
3 & 4	0.009	0.059	0.010
5 & 6	0.017	0.082	0.017
7	0.194	0.043	0.193
8	0.169	0.018	0.168

Table 9: Component importance using gamma distributions. Variance of component 1 lifetimes: $1.316 \cdot 10^5$.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.213	0.248	0.213
2	0.420	0.415	0.420
3 & 4	0.008	0.058	0.009
5 & 6	0.015	0.081	0.015
7	0.166	0.042	0.164
8	0.156	0.017	0.155

Table 10: Component importance using gamma distributions. Variance of component 1 lifetimes: $2.924 \cdot 10^5$.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.301	0.248	0.300
2	0.375	0.414	0.376
3 & 4	0.007	0.058	0.008
5 & 6	0.013	0.081	0.014
7	0.149	0.049	0.148
8	0.134	0.018	0.133

Table 11: Component importance using gamma distributions. Variance of component 1 lifetimes: $1.170 \cdot 10^6$.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.476	0.239	0.475
2	0.279	0.421	0.280
3 & 4	0.006	0.058	0.006
5 & 6	0.010	0.082	0.010
7	0.111	0.043	0.111
8	0.102	0.017	0.101

Table 12: The ranks of the extended component importance according to the results given in Tables 7, 8, 9, 10 and 11.

Table	Rank
7	$2 > 7 > 8 > 1 > 5 \approx 6 > 3 \approx 4$
8	$2 > 7 > 8 > 1 > 5 \approx 6 > 3 \approx 4$
9	$2 > 1 > 7 > 8 > 5 \approx 6 > 3 \approx 4$
10	$2 > 1 > 7 > 8 > 5 \approx 6 > 3 \approx 4$
11	$1 > 2 > 7 > 8 > 5 \approx 6 > 3 \approx 4$

5 CONCLUDING REMARKS

In the present paper first a review of basic ideas, concepts and theoretical results, as treated in (Natvig and Gåsemyr 2008), for the Natvig measures of component importance for repairable systems, and its extended version, has been given. The theory was then applied to an offshore oil and gas production system which is said to be functioning if it can produce some amount of both oil and gas. First life- and repair times are assumed to be exponentially distributed and then gamma distributed both in accordance with the data given in the memo (Signoret and Clave 2007). The time horizon is set at 100000 days so stationarity is reached.

A finding from the simulations of this case study is that the results for the original Natvig measure and its extended version, also taking a dual term into account, are almost identical. This is perfectly sensible since the dual term vanishes because the fictive prolonged repair times are much shorter than the fictive prolonged lifetimes. The weaknesses of this system are linked to the lifetimes and not the repair times.

Component 1 is the well being in series with the rest of the system. For this component we see that the extended component importance, in the gamma case is increasing with increasing variances, and decreasing shape parameters, all greater than 1, in the lifetime distribution. This is in accordance with a theoretical result for the Weibull distribution. Along with this increased importance we also observe a corresponding improvement in its ranking.

As a conclusion we feel that the presented Natvig measures of component importance for repairable systems on the one hand represent a theoretical novelty. On the other hand the case study indicates a great potential for applications, especially due to the simulation methods developed, as presented in the companion paper (Huseby, Eide, Isaksen, Natvig, and Gåsemyr 2008).

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