Introduction to repairable systems
STK4400 – Spring 2011

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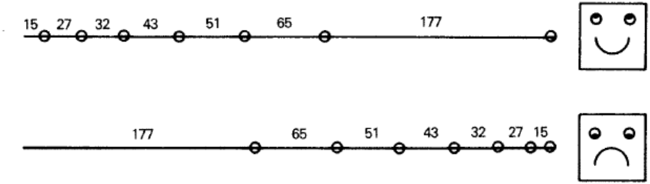
Definition of repairable system

Ascher and Feingold (1984):

"A repairable system is a system which, after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method, other than replacement of the entire system".

Ascher and Feingold's mission in 1984

Ascher and Feingold presented the following example of a "happy" and "sad" system:

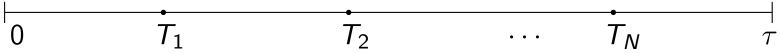


Their claim:

Reliability engineers do not recognize the difference between these cases since they always treat times between failures as i.i.d. and fit probability models like Weibull.

Use nonstationary stochastic point process models to analyze repairable systems data!

Today: Recurrent events extensively studied

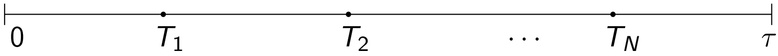


- Observe events occurring in time
- Applications: engineering and reliability studies, public health, clinical trials, politics, finance, insurance, sociology, etc.

Reliability applications:

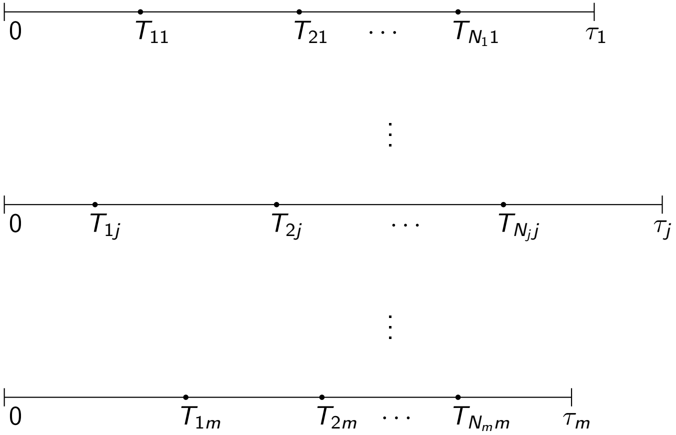
- breakdown or failure of a mechanical or electronic system
- discovery of a bug in an operating system software
- the occurrence of a crack in concrete structures
- the breakdown of a fiber in fibrous composites
- Warranty claims of manufactured products

Important aspects for modelling and analysis



- Trend in times between events?
- Renewals at events?
- "Randomness" of events?
- Dependence on covariates?
- Unobserved heterogeneity ("frailty", "random effects") among individual processes?
- Dependence between event process and the censoring at τ?

Typical data format

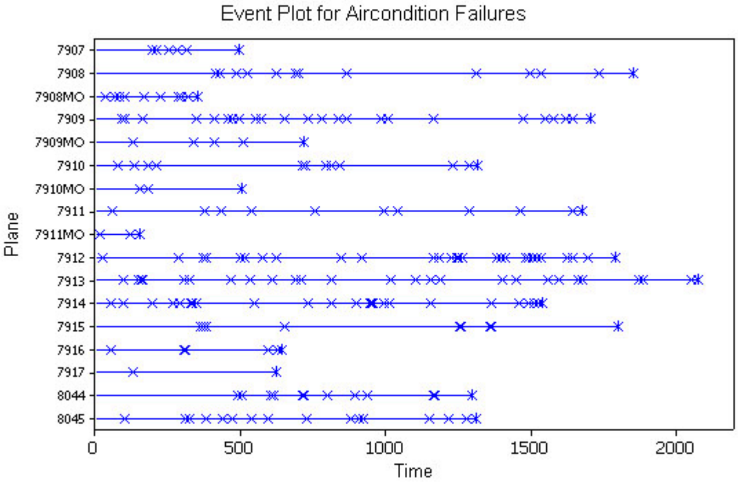


Covariates if available (fixed or time-varying):

$$X_j(t);\ j = 1, 2, \ldots, m$$

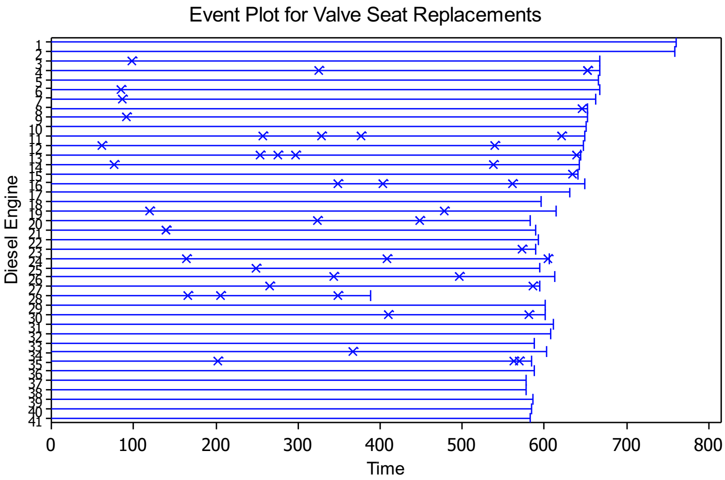
Proschan (1963): The classical "aircondition data"

Times of failures of aircondition system in a fleet of Boeing 720 airplanes

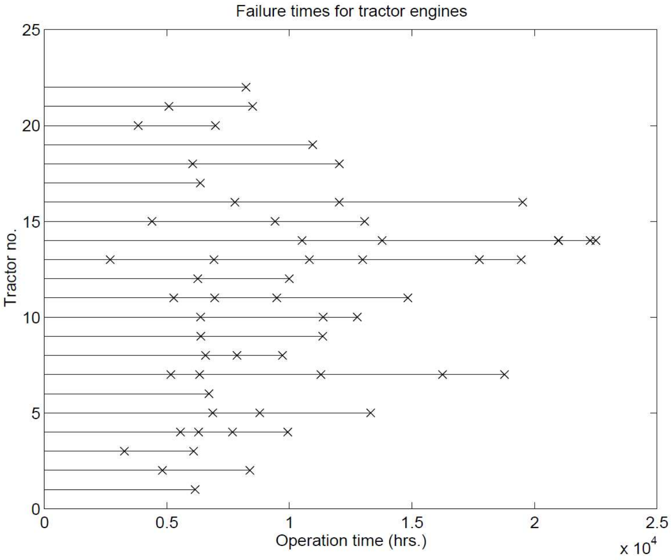


Nelson (1995): Valve seat data

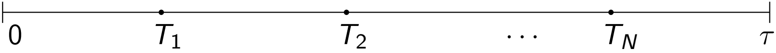
Times of valve-seat replacements in a fleet of 41 diesel engines



Barlow and Davis (1977): Tractor data



Basic models for repairable systems



- RP(F): Renewal process with interarrival distribution F.

  Defining property:

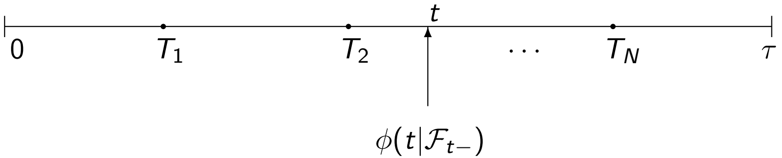
  - Times between events are i.i.d. with distribution F

- NHPP($\lambda(\cdot)$): Nonhomogeneous Poisson process with intensity $\lambda(t)$.

  Defining property:

  1. Number of events in $(0, t]$ is Poisson-distributed with expectation $\int_0^t \lambda(u)du$
  2. Number of events in disjoint time intervals are stochastically independent

Point process modelling of recurrent event processes



- $\mathcal{F}_{t-}$ = history of events until time t.
- Conditional intensity at t given history until time t,

$$\phi(t|\mathcal{F}_{t-}) = \lim_{\Delta t \downarrow 0} \frac{Pr(\text{failure in } [t, t+\Delta t)|\mathcal{F}_{t-})}{\Delta t}$$

Special cases: The basic models

- NHPP($\lambda(\cdot)$):

$$\phi(t|\mathcal{F}_{t-}) = \lambda(t)$$

so conditional intensity is independent of history.
Interpreted as "minimal repair" at failures

- RP(F) (where F has hazard rate $z(\cdot)$):

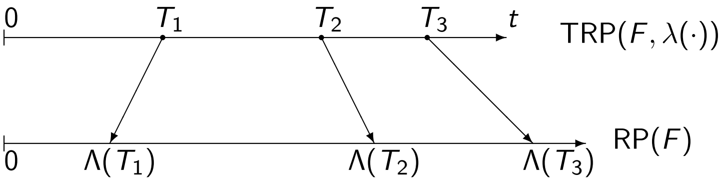
$$\phi(t|\mathcal{F}_{t-}) = z(t - T_{N(t-)})$$

so conditional intensity depends (only) on time since last event.
Interpreted as "perfect repair" at failures

- Between minimal and perfect repair? *Imperfect repair* models.

Trend Renewal Process – TRP
(BL, Elvebakk and Heggland 2003)

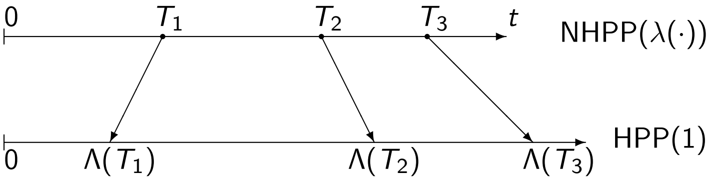
- Trend function: $\lambda(t)$ (cumulative $\Lambda(t) = \int_0^t \lambda(u)du$)
- Renewal distribution: F with expected value 1 (for uniqueness)



SPECIAL CASES:

- NHPP: F is standard exponential distribution
- RP: $\lambda(t)$ is constant in t

Motivation for TRP: Well known property of NHPP



Point process formulation of TRP:

Conditional intensity of TRP$(F, \lambda(\cdot))$:

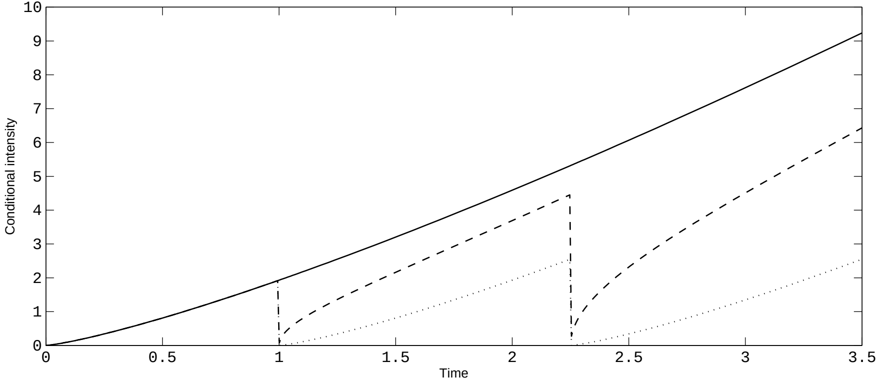
$$\phi(t|\mathcal{F}_{t-}) = z(\Lambda(t) - \Lambda(T_{N(t-)}))\lambda(t)$$

where $z(\cdot)$ is hazard rate of F

Recall special cases:

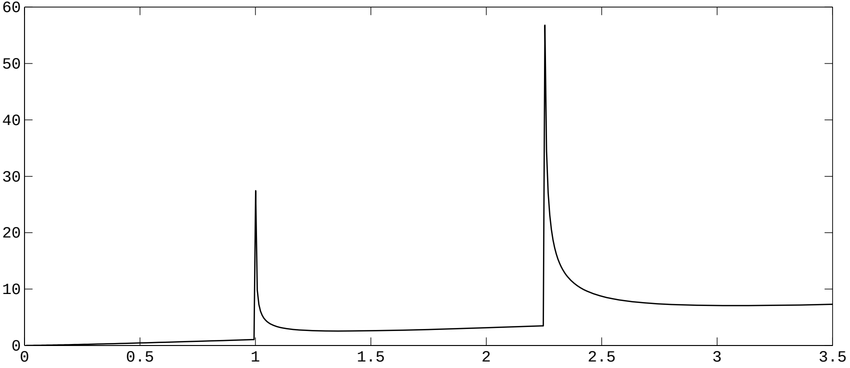
- NHPP: $z(\cdot) \equiv 1$, implies $\phi(t|\mathcal{F}_{t-}) = \lambda(t)$
- RP: $\lambda(\cdot) \equiv 1$, implies $\phi(t|\mathcal{F}_{t-}) = z(t - T_{N(t-)})$

Comparison of NHPP, TRP and RP: Conditional intensities



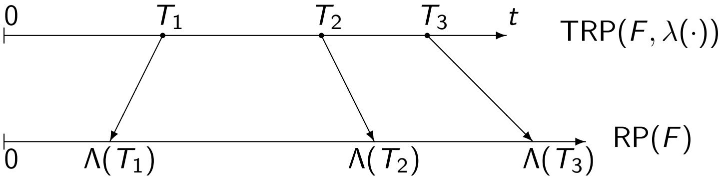
Example: Failures observed at times 1.0 and 2.25.
Conditional intensities for NHPP (solid); TRP (dashed); RP (dotted)

Conditional intensities of TRP with DFR renewal distribution and increasing trend function



Failures observed at times 1.0 and 2.25.
Renewal: $F \sim$ Weibull, shape 0.5
Trend: $\lambda(t) = t^2$

Interpretation of $\Lambda(t)$



By using results from renewal theory:

$$\lim_{t \to \infty} \frac{N(t)}{\Lambda(t)} = 1 \; (a.s.)$$

$$\lim_{t \to \infty} \frac{E(N(t))}{\Lambda(t)} = 1$$

since F is assumed to have expected value 1.

Alternative models: Cox' modulated renewal model

- Cox (1972):

$$\phi(t|\mathcal{F}_{t-}) = z(t - T_{N(t-)})\lambda(t)$$

- Compare to TRP:

$$\phi(t|\mathcal{F}_{t-}) = z(\Lambda(t) - \Lambda(T_{N(t-)}))\lambda(t)$$

Lawless and Thiagarajah (1996):

$$\phi(t|\mathcal{F}_{t-}) = e^{\theta' h(t)}$$

where $h(t)$ is a vector of observable functions, depending on t and possibly also on the history $\mathcal{F}_{t-}$, and θ is a vector of parameters.

This model allows the use of various *dynamic covariates*, such as

- number of previous failures of the system, $N(t-)$
- time since last event, $t - T_{N(t-)}$

Alternative models: The model by Pena and Hollander

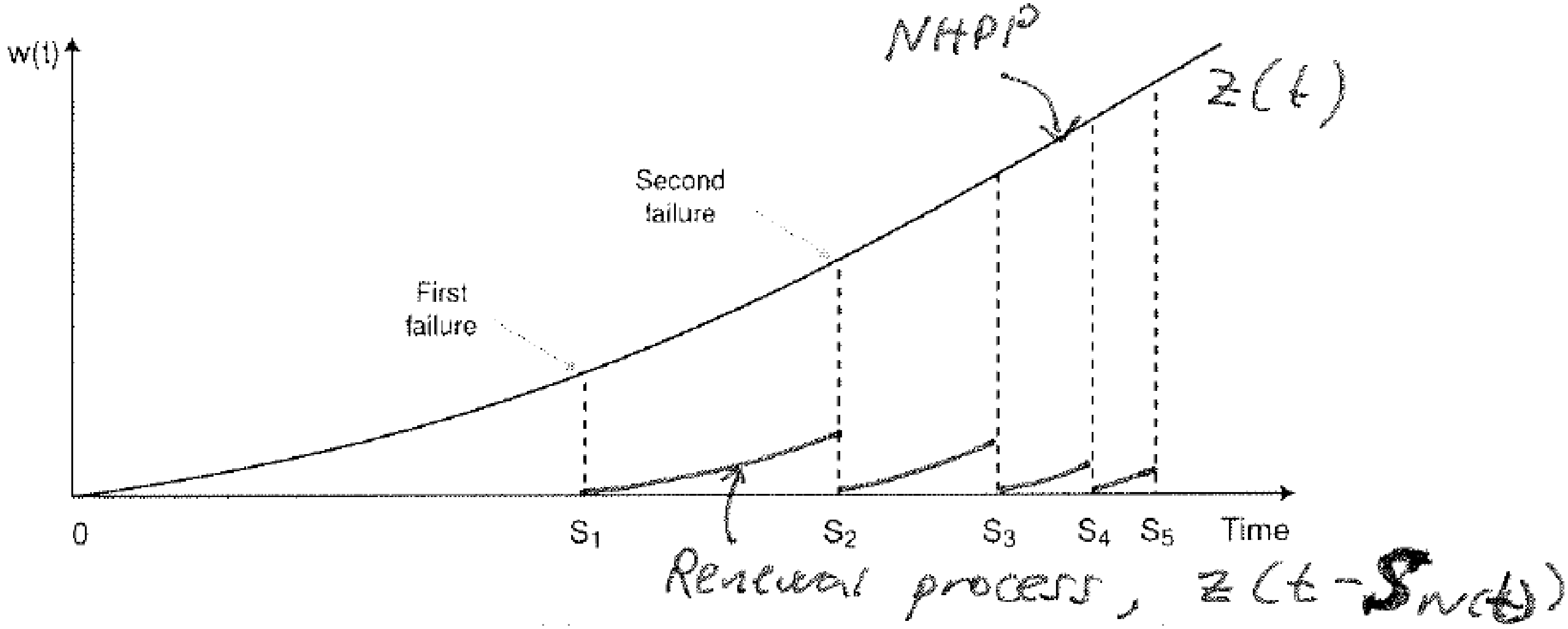
Pena and Hollander (2004):

$$\phi(t|\mathcal{F}_{t-}, a) = a\, \lambda_0(\mathcal{E}(t))\, \rho(N(t-); \alpha)\, g(x(t), \beta)$$

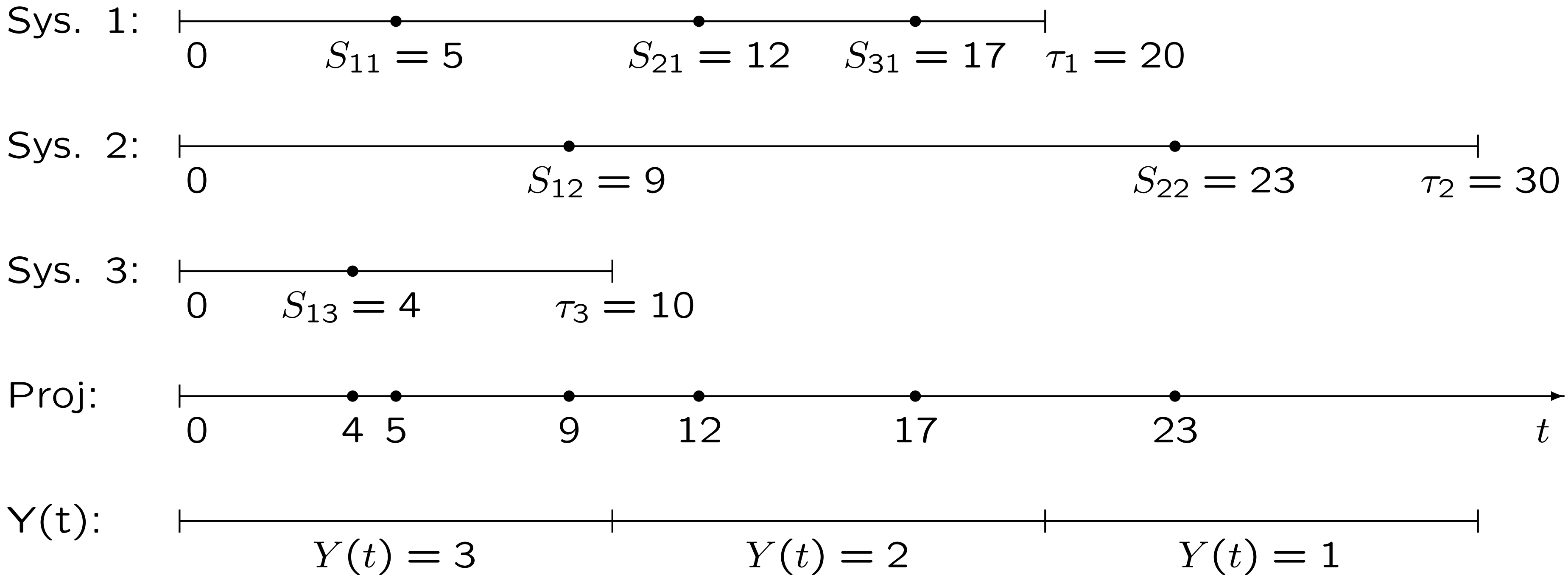
where

- a is a positive unobserved random variable (frailty)
- $\lambda_0(\cdot)$ is a baseline hazard rate function
- $\mathcal{E}(t)$ is "effective age" process, a predictable process modeling impact of performed interventions after each event. Possible choices
  - Minimal intervention on repair: $\mathcal{E}(t) = t$
  - Perfect intervention on repair: $\mathcal{E}(t) = t - T_{N(t-)}$
  - Imperfect repair: $\mathcal{E}(t) =$ time since last perfect repair
  - Age reduction models: $\mathcal{E}(t)$ is reduced by a certain factor at each repair
- $\rho(\cdot; \cdot)$ can for example be on geometric form, $\rho(k; \alpha) = \alpha^k$
- $g(x(t), \beta)$ models impact of covariates, for example on Cox-form

CONDITIONAL ROCOF BY MINIMAL REPAIR (NHPP) AND PERFECT REPAIR (RENEWAL PROCESS)



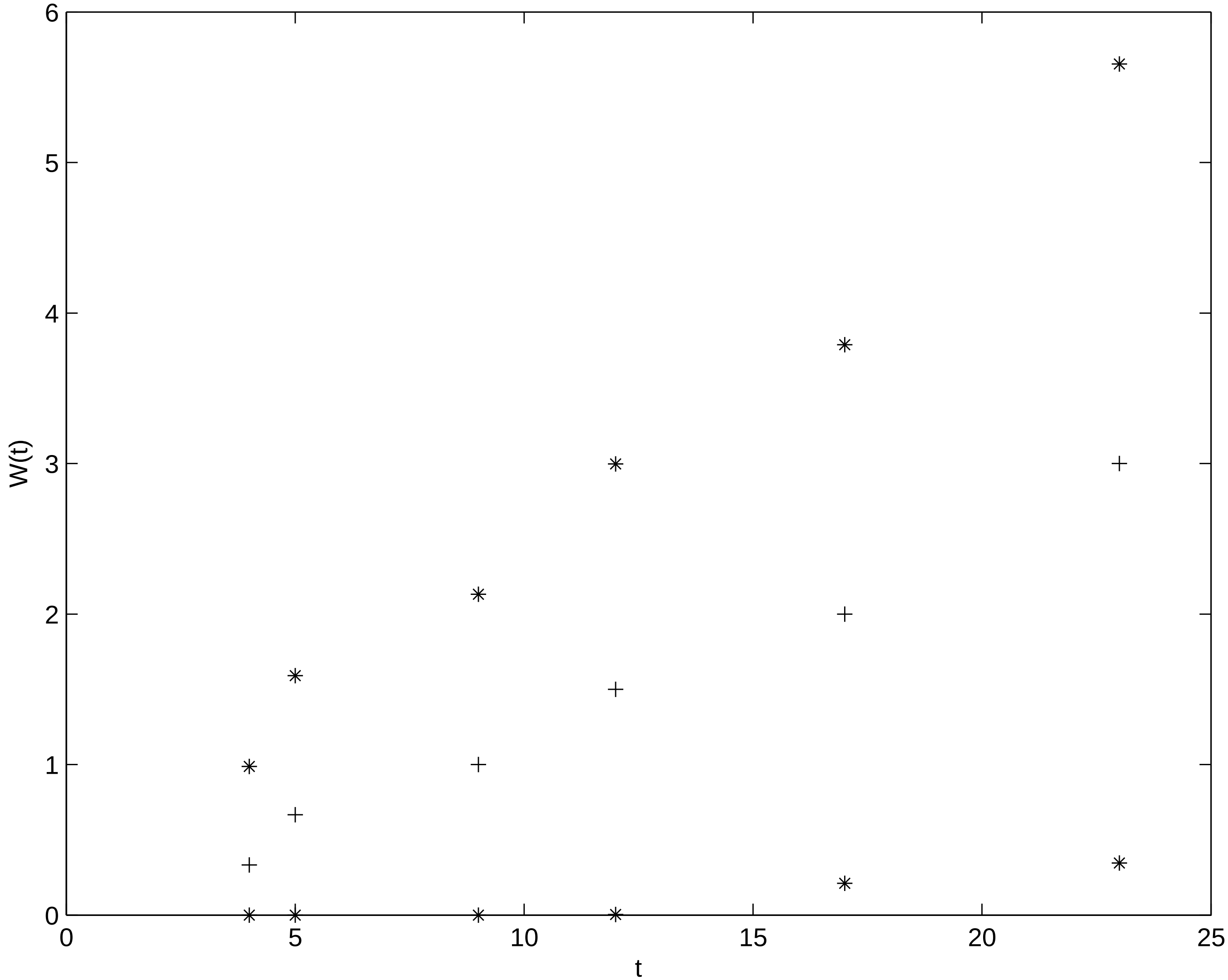
SIMPLE EXAMPLE WITH THREE SYSTEMS



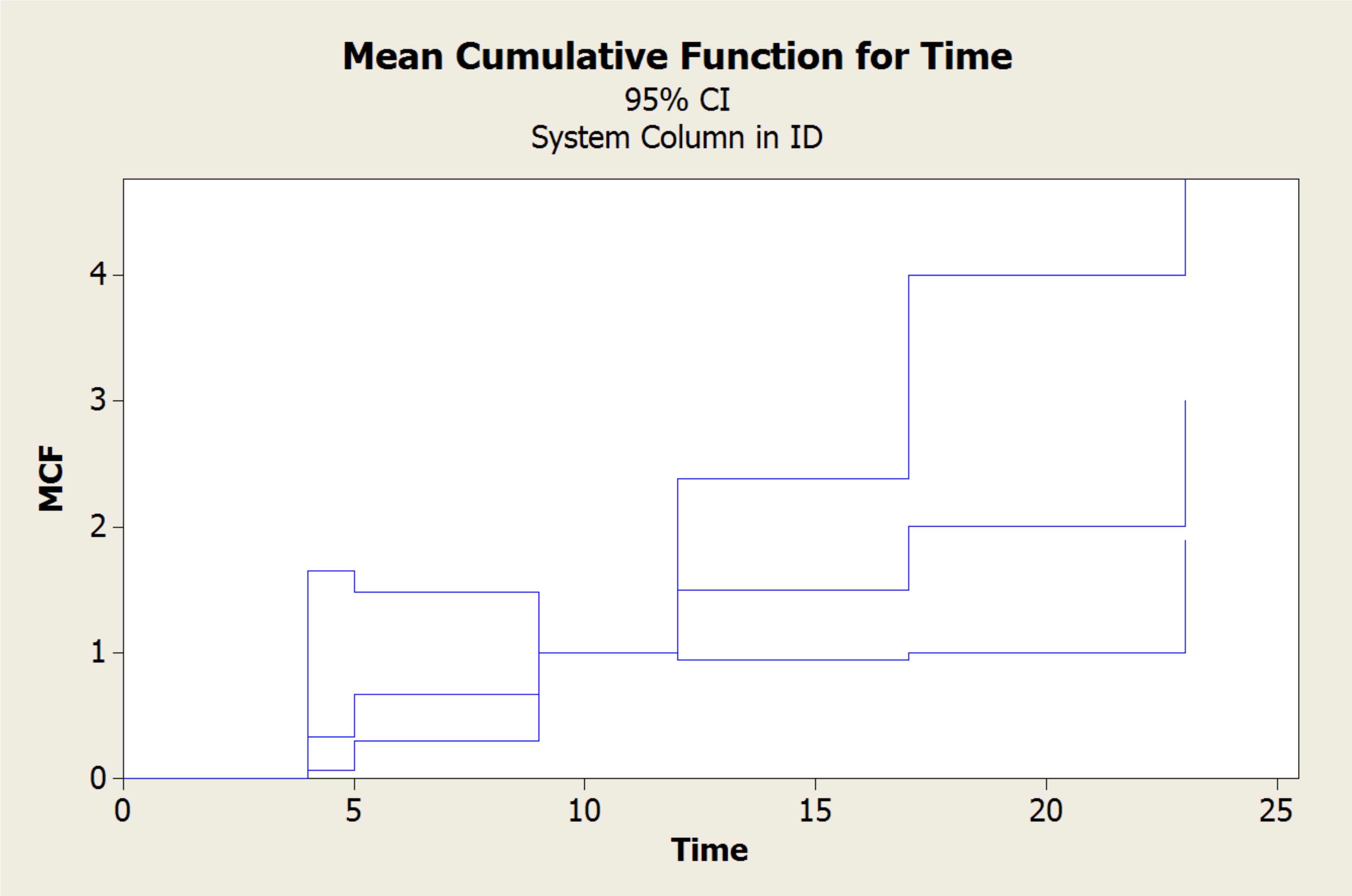
COMPUTATIONS FOR THE NELSON-AALEN ESTIMATOR

t	$1/Y(t)$	$1/Y(t)^2$	$\widehat{W}(t)$	$\widehat{Var\widehat{W}}(t)$	$\widehat{SD\widehat{W}}(t)$
4	1/3	1/9	1/3	1/9	0.3333
5	1/3	1/9	2/3	2/9	0.4714
9	1/3	1/9	1	1/3	0.5774
12	1/2	1/4	3/2	7/12	0.7638
17	1/2	1/4	2	5/6	0.9129
23	1	1	3	11/6	1.3540

ESTIMATED $W(t)$ with 95% confidence limits (Nelson-Aalen)



Simple Example With 3 Systems



Nelson-Aalen estimator for Cumulative ROCOF $W(t)$

1. Order all failure times as $t_1 < t_2 < \ldots t_n$.

2. Let $d_j(t_i) = \#$ events in system j at t_i.

3. Let $d(t_i) = \sum_{j=1}^{m} d_j(t_i) = \#$ events in all systems at t_i.

4. Let $Y_j(t) = \begin{cases} 1 & \text{if system } j \text{ is under observation at time } t \\ 0 & \text{otherwise} \end{cases}$

5. Let $Y(t) = \sum_{j=1}^{m} Y_j(t) = \#$ systems under observation at time t .

Then

Under general assumptions: $\widehat{W}(t) = \sum_{t_i \le t} \frac{d(t_i)}{Y(t_i)}$.

Assuming NHPP: $\widehat{\text{Var}\,\widehat{W}(t)} = \sum_{t_i \le t} \frac{d(t_i)}{\{Y(t_i)\}^2}$

Under general assumptions (MINITAB): $\widehat{\text{Var}\,\widehat{W}(t)} = \sum_{j=1}^{m} \left\{ \sum_{t_i \le t} \frac{Y_j(t_i)}{Y(t_i)} \left[d_j(t_i) - \frac{d(t_i)}{Y(t_i)} \right] \right\}^2$

Illustration of last formula for Simple NHPP Example (Compare with MINITAB Output):

$$
\begin{aligned}
\text{Var}\,\widehat{\widehat{W}}(4) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2+\left\{\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2+\left\{\frac{1}{3}\left[1-\frac{1}{3}\right]\right\}^2 \\
&= \frac{6}{81}=0.2722^2
\end{aligned}
$$

$$
\begin{aligned}
\text{Var}\,\widehat{\widehat{W}}(5) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&= \frac{6}{81}=0.2722^2
\end{aligned}
$$

$$\begin{aligned}
\text{Var}\,\widehat{\widehat{W}}(9) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\text{Var}\,\widehat{\widehat{W}}(12) &= \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{2}\left[1-\frac{1}{2}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{2}\left[0-\frac{1}{2}\right]\right\}^2 \\
&+ \left\{\frac{1}{3}\left[1-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]+\frac{1}{3}\left[0-\frac{1}{3}\right]\right\}^2 \\
&= \frac{1}{8} = 0.3536^2
\end{aligned}$$

Simple Example With 3 Systems

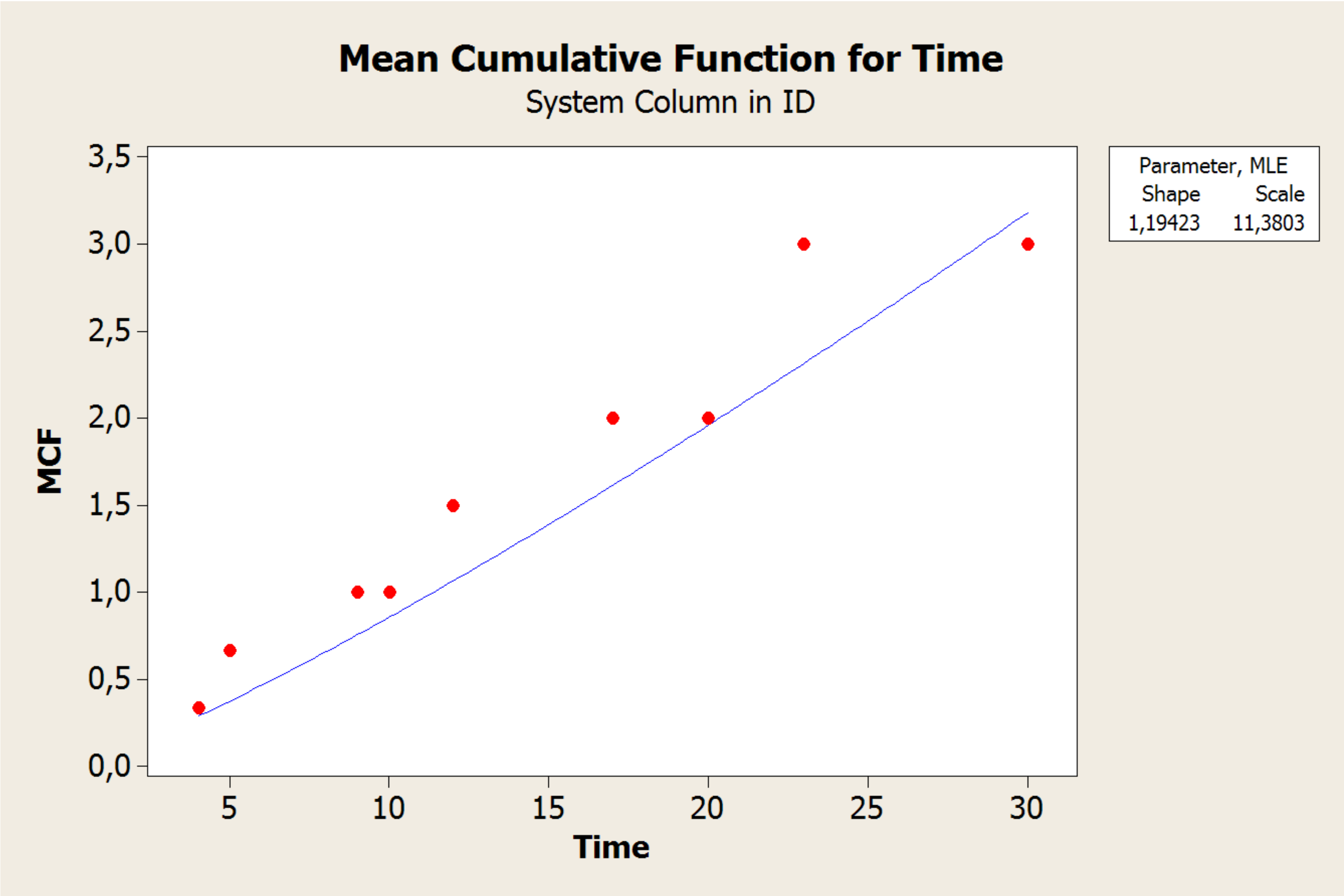
Power Law NHPP Model: $W(t;\alpha,\theta) = (t/\theta)^{\alpha}$

Results for: SimpleNHPP.MTW

Parametric Growth Curve: Time

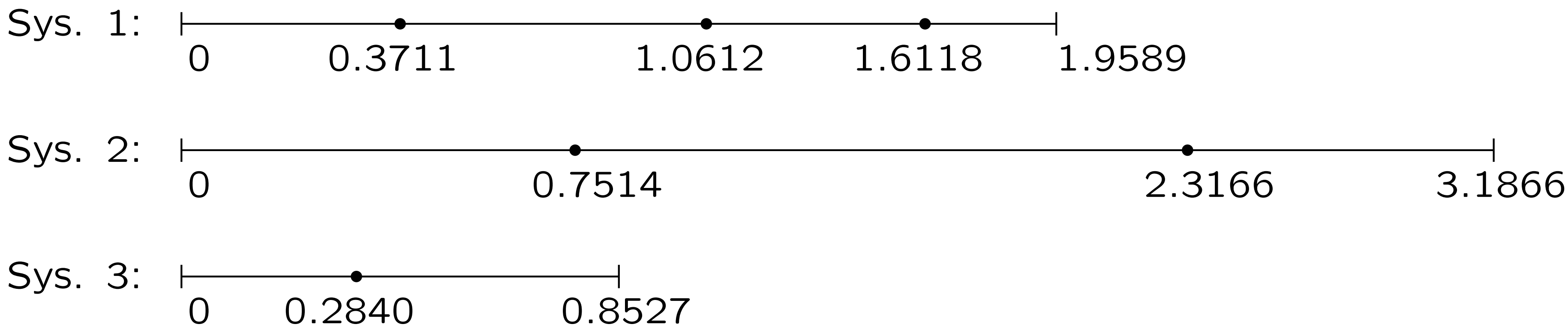
```
System:  ID

Model: Power-Law Process
Estimation Method: Maximum Likelihood


Parameter Estimates

                     Standard    95% Normal CI
Parameter  Estimate     Error      Lower     Upper
Shape       1,19423     0,445  0,323015   2,06545
Scale       11,3803     4,840   1,89335   20,8672


Test for Equal Shape Parameters
Bartlett's Modified Likelihood Ratio Chi-Square

Test Statistic   0,06
P-Value         0,972
DF                  2


Trend Tests

                  MIL-Hdbk-189        Laplace's
                TTT-based  Pooled  TTT-based  Pooled  Anderson-Darling
Test Statistic       9,03    8,89       0,28    0,31              0,28
P-Value             0,599   0,576      0,781   0,756             0,954
DF                     12      12
```

Simple Example With 3 Systems

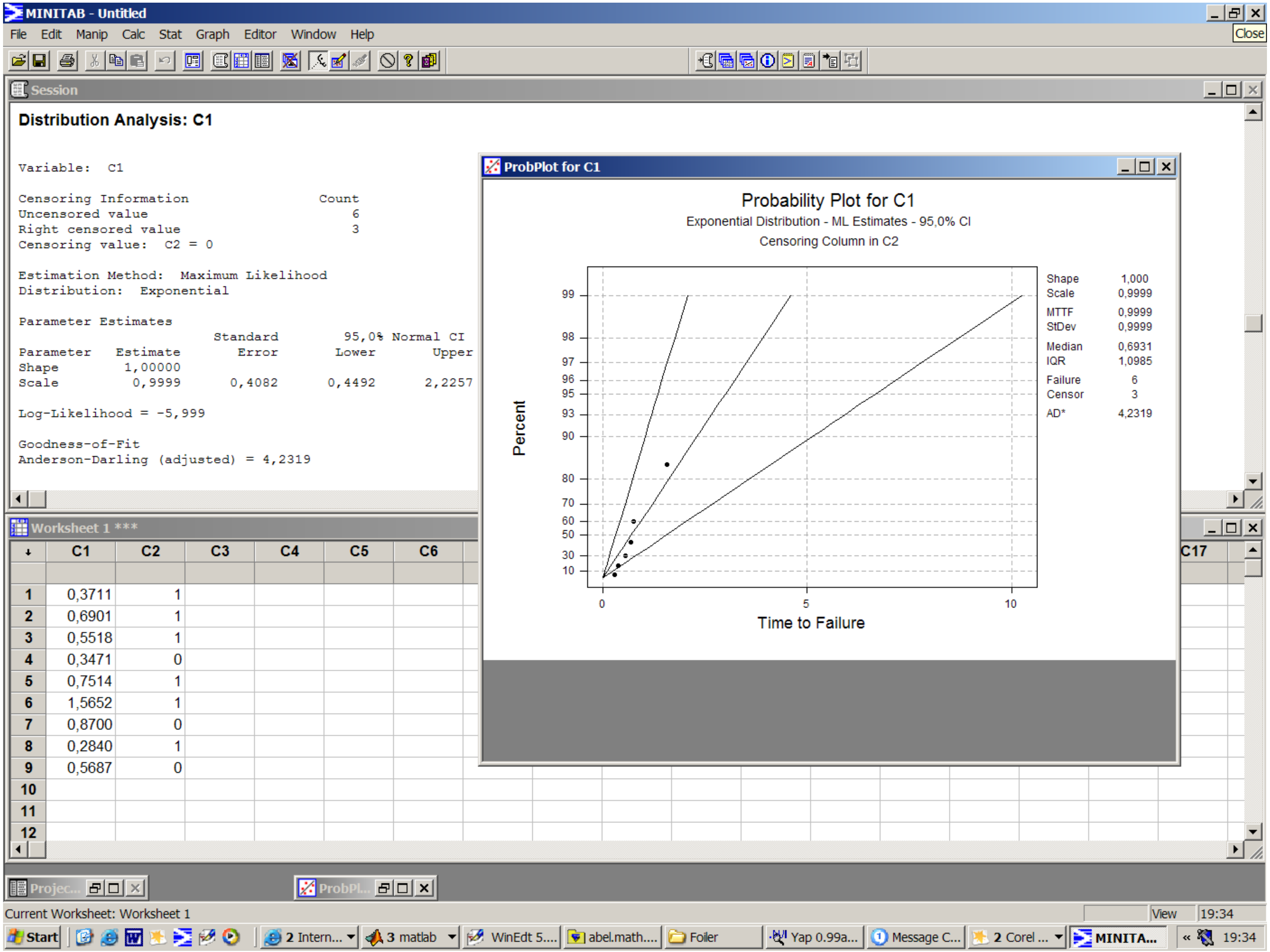


RESIDUAL PROCESS: "SIMPLE EXAMPLE".
Data points (and endpoints on axes) are transformed with the estimated cumulative ROCOF,

$$\widehat{W}(t) = 0.0538 \cdot t^{1.20}$$



Times between events, plus censored times at the end of each axis, are on the next slide anlysed by MINITAB as a set of censored exponential variables.


MINITAB - Untitled
File Edit Manip Calc Stat Graph Editor Window Help
Session
Distribution Analysis: C1
Variable: C1
Censoring Information Count
Uncensored value 6
Right censored value 3
Censoring value: C2 = 0
Estimation Method: Maximum Likelihood
Distribution: Exponential
Parameter Estimates
Standard 95,0% Normal CI
Parameter Estimate Error Lower Upper
Shape 1,00000
Scale 0,9999 0,4082 0,4492 2,2257
Log-Likelihood = -5,999
Goodness-of-Fit
Anderson-Darling (adjusted) = 4,2319
Worksheet 1 ***
C1 C2 C3 C4 C5 C6
1 0,3711 1
2 0,6901 1
3 0,5518 1
4 0,3471 0
5 0,7514 1
6 1,5652 1
7 0,8700 0
8 0,2840 1
9 0,5687 0
ProbPlot for C1
Probability Plot for C1
Exponential Distribution - ML Estimates - 95,0% CI
Censoring Column in C2
Percent
Time to Failure
Shape 1,000
Scale 0,9999
MTTF 0,9999
StDev 0,9999
Median 0,6931
IQR 1,0985
Failure 6
Censor 3
AD* 4,2319
Current Worksheet: Worksheet 1

Valve Seat Replacement Times
(Nelson and Doganaksoy 1989)

Data collected from valve seats from a fleet of 41 diesel engines (days of operation)

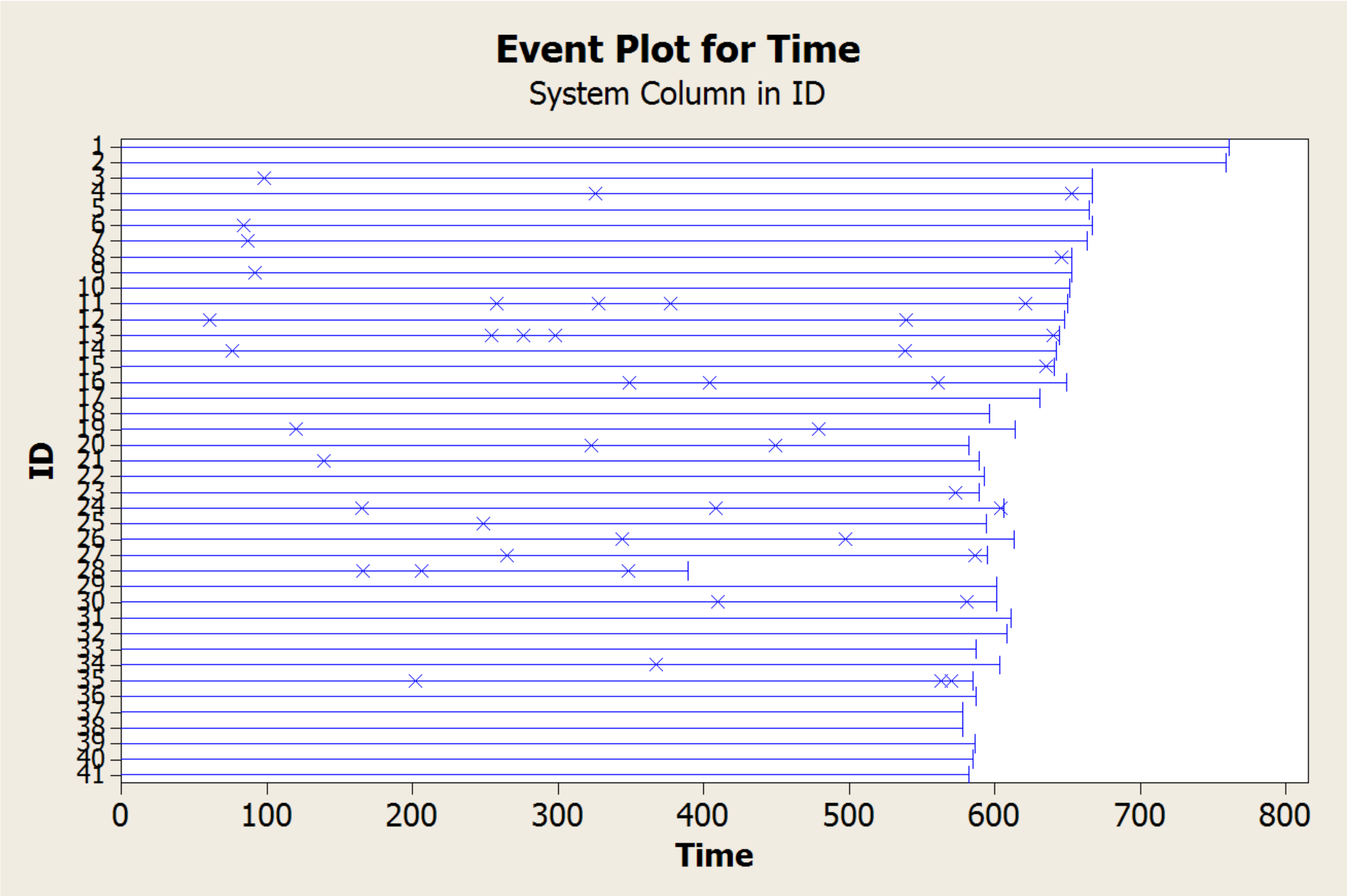
- Each engine has 16 valves
- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

VALVESEAT DATA

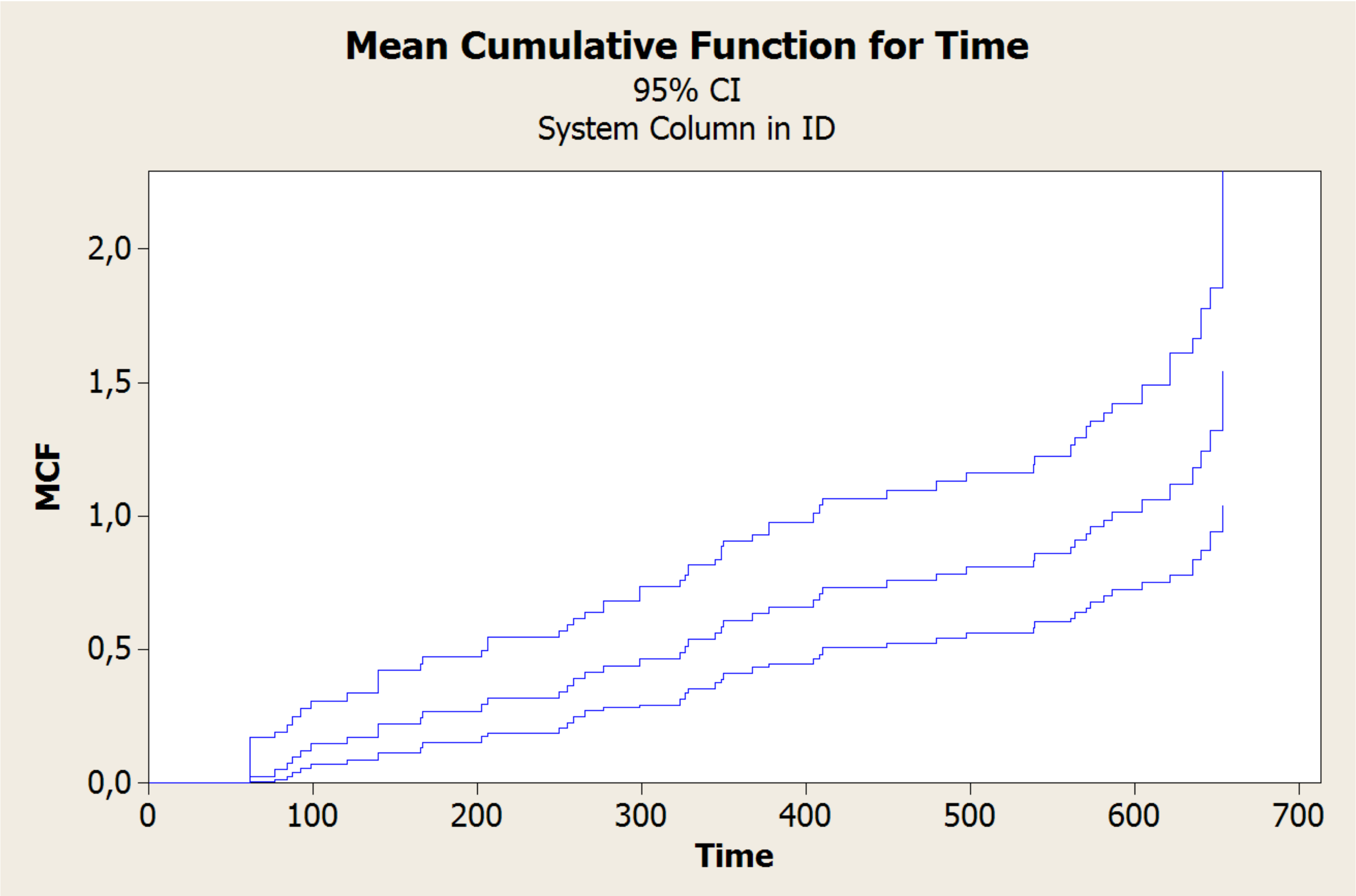
TMA4275valveseat.MTW ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C
	ID	Time							
1	1	761							
2	2	759							
3	3	98							
4	3	667							
5	4	326							
6	4	653							
7	4	653							
8	4	667							
9	5	665							
10	6	84							
11	6	667							
12	7	87							
13	7	663							
14	8	646							
15	8	653							
16	9	92							
17	9	653							
18	10	651							
19	11	258							
20	11	328							
21	11	377							
22	11	621							
23	11	650							
24	12	61							
25	12	539							
26	12	648							

VALVESEAT DATA



VALVESEAT DATA



Times of Unscheduled Maintenance Actions for a USS Grampus Diesel Engine

- Unscheduled maintenance actions caused by failure of imminent failure.

- Unscheduled maintenance actions are inconvenient and expensive.

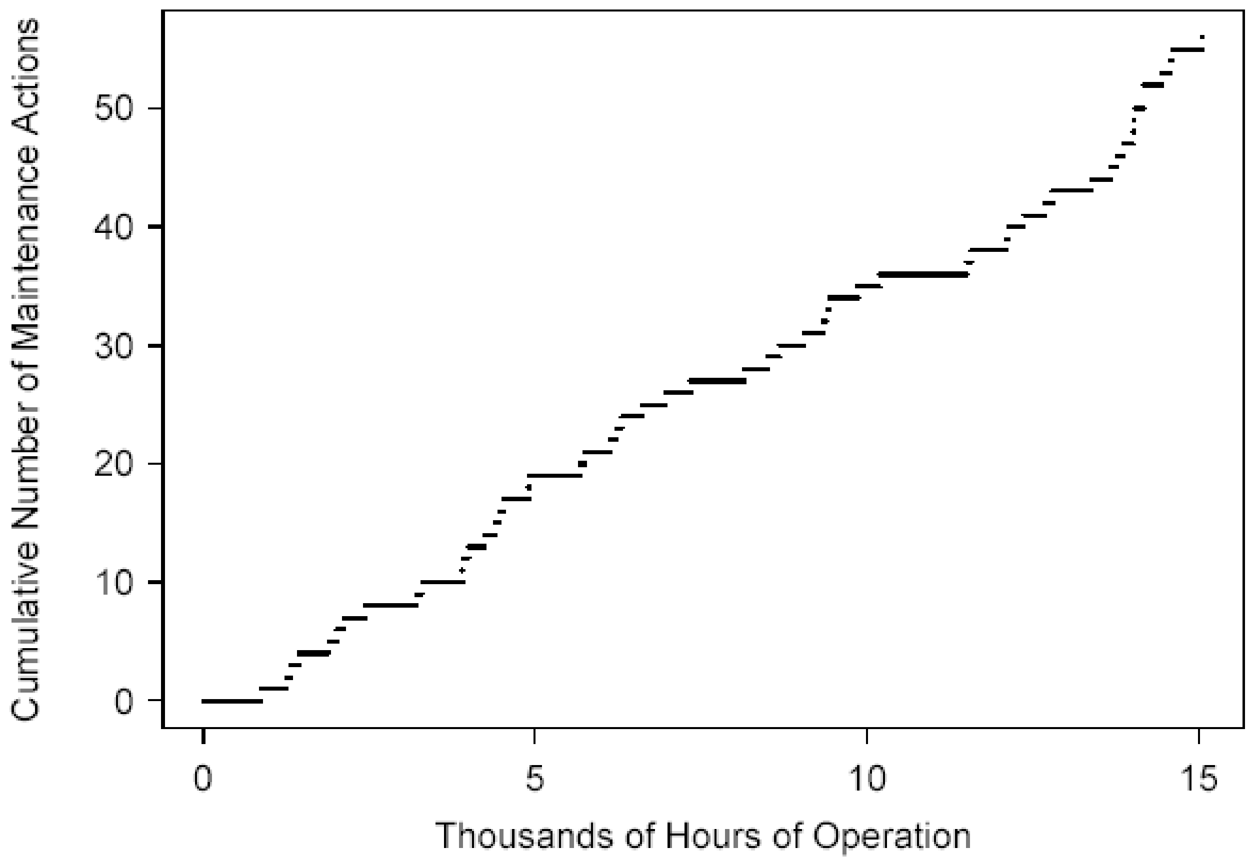
- Data available for 16,000 operating hours.

- Data from Lee (1980).

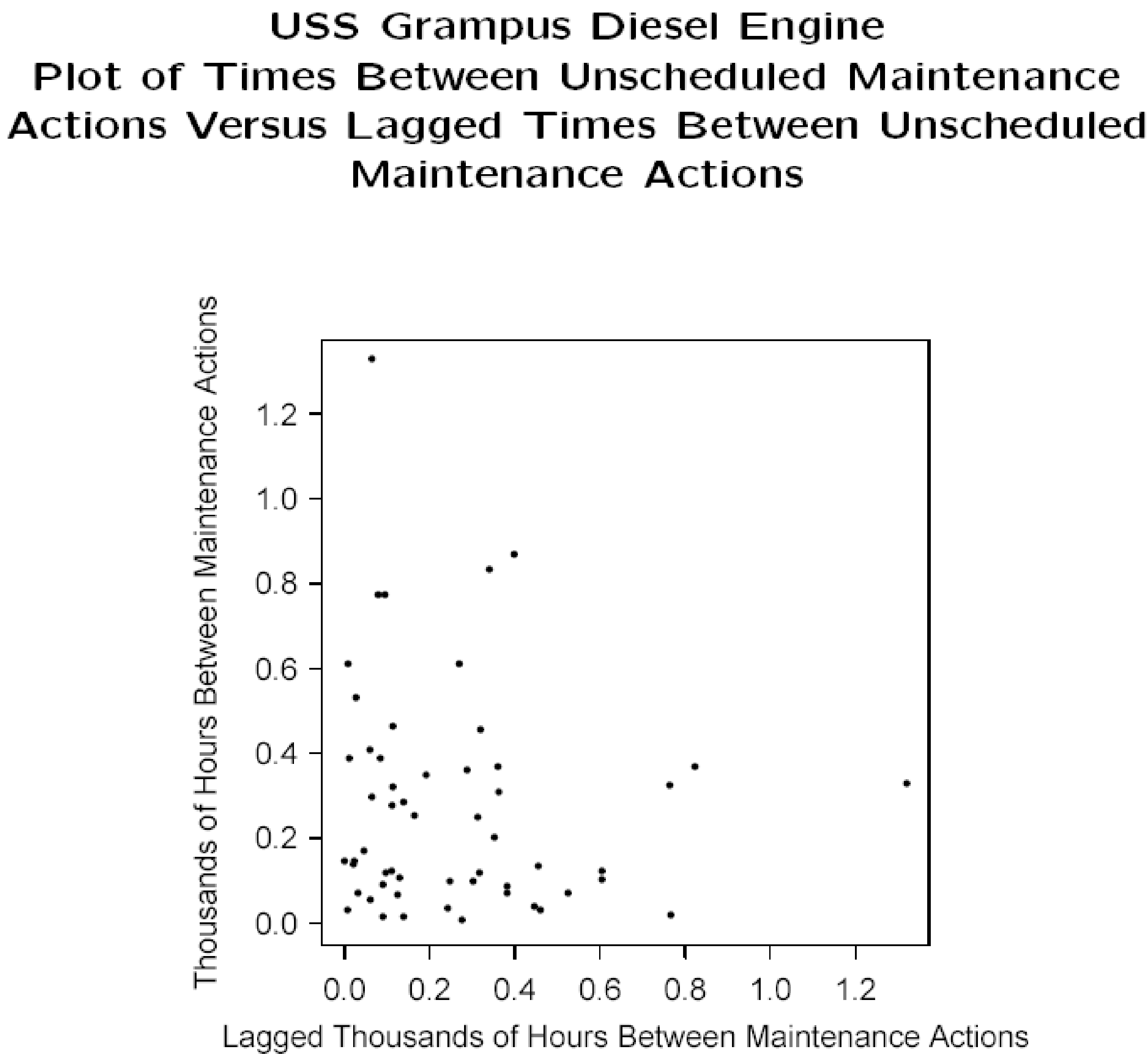
- Is the system deteriorating (i.e., are failures occurring more rapidly as the system ages)?

- Can the occurrence of unscheduled maintenance actions be modeled by an HPP?

Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours for a USS Grampus Diesel Engine
Lee (1980)



Grampus- data: Plot of (T_i, T_{i+1}) to investigate whether times between failures can be assumed independent. The figure does not indicate a correlation between successive times.



The Likelihood for the NHPP - Single Unit

- With **interval** recurrence data.

  Suppose that the unit has been observed for a period $(0, t_a]$ and the data are the number of recurrences $d_1, \ldots, d_m$ in the nonoverlapping intervals $(t_0, t_1]$, $(t_1, t_2]$, $\ldots$, $(t_{m-1}, t_m]$ (with $t_0 = 0$, $t_m = t_a$).

$$
\begin{aligned}
L(\boldsymbol{\theta}) &= \Pr\left[N(t_0, t_1) = d_1, \ldots, N(t_{m-1}, t_m) = d_m\right] \\
&= \prod_{j=1}^{m} \Pr\left[N(t_{j-1}, t_j) = d_j\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \exp\left[-\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \times \exp\left[-\mu(t_0, t_a; \boldsymbol{\theta})\right]
\end{aligned}
$$

The Likelihood for the NHPP (Continued)

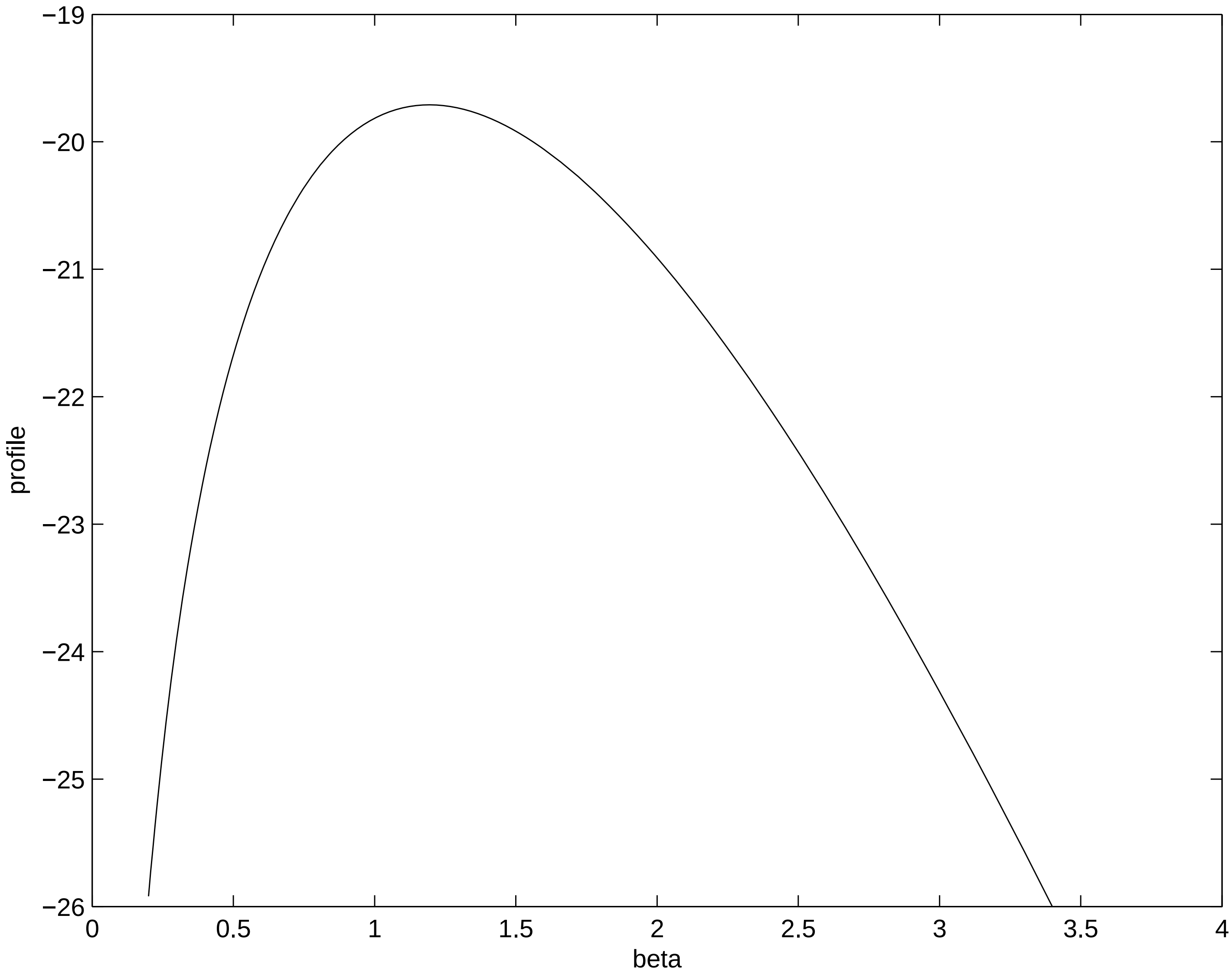
- If the number of intervals m increases and there are **exact** recurrences at $t_1 \leq \ldots \leq t_r$ (here $r = \sum_{j=1}^{m} d_j$, $t_0 \leq t_1$, $t_r \leq t_a$), then using a limiting argument it follows that the likelihood in terms of the density approximation is

$$L(\boldsymbol{\theta}) = \prod_{j=1}^{r} \nu(t_j; \boldsymbol{\theta}) \times \exp\left[-\mu(0, t_a; \boldsymbol{\theta})\right]$$

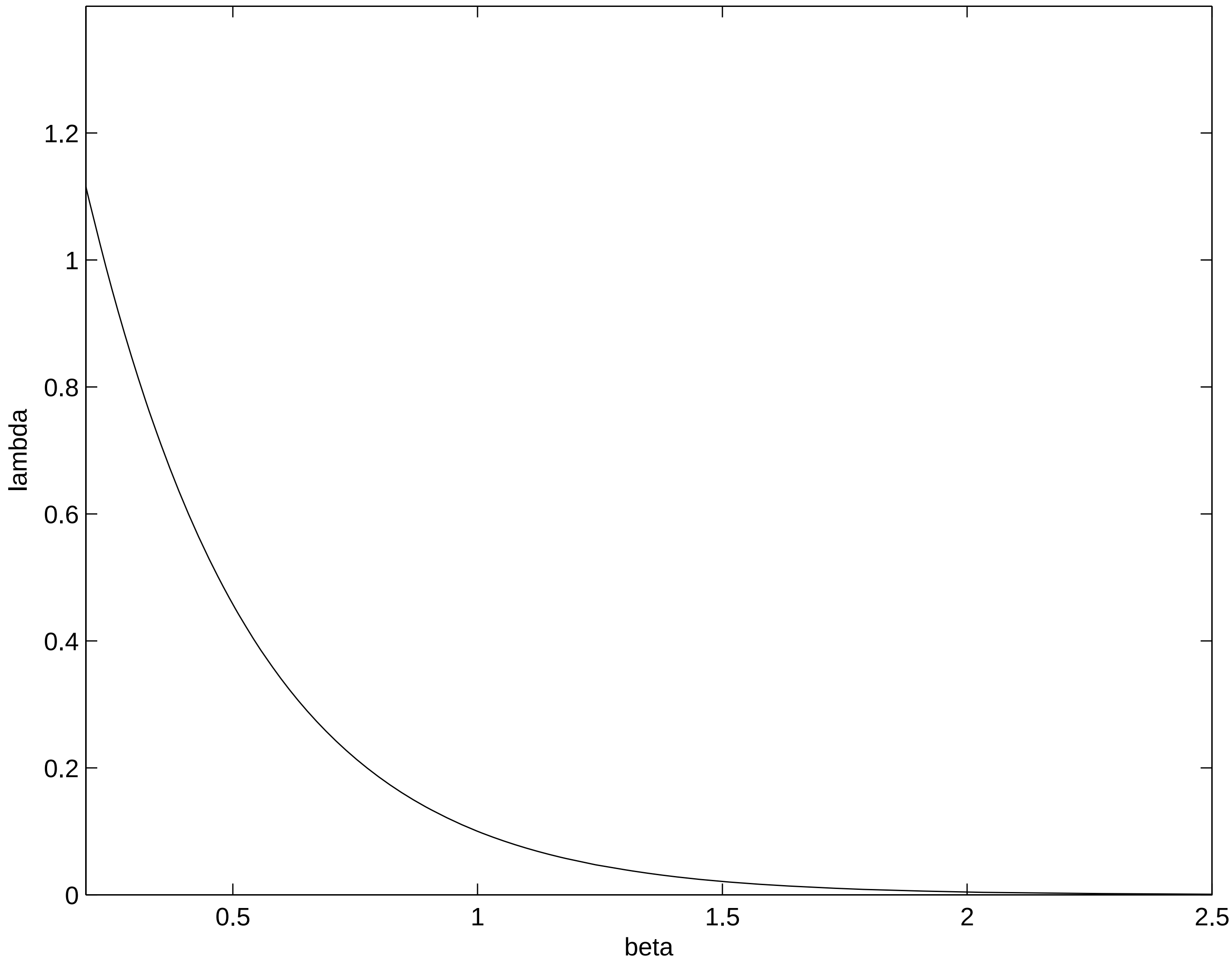
- For simplicity, above we assumed that the intervals are contiguous. Obvious changes to the formula above give the likelihood when there are gaps among the intervals.

- In both cases (the interval data or exact recurrences data) the same methods used in Chapters 7, 8 can be used to obtain the ML estimate $\widehat{\boldsymbol{\theta}}$ and confidence regions for $\boldsymbol{\theta}$ or functions of $\boldsymbol{\theta}$.

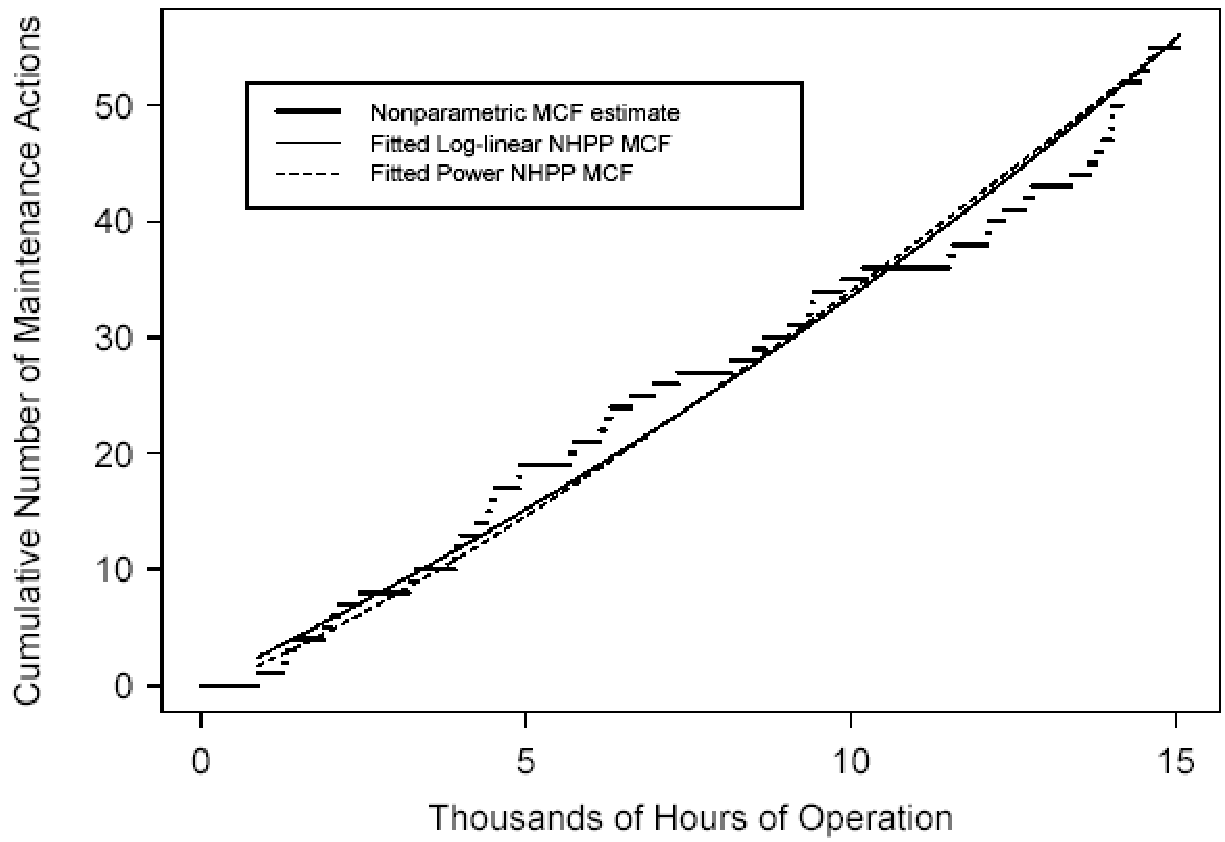
PROFILE LIKELIHOOD FOR BETA
("SIMPLE EXAMPLE")
$\widehat{\beta} = 1.20,\ \widehat{\lambda} = 0.0538.$



CONNECTION BETWEEN LAMBDA OG BETA ("SIMPLE EXAMPLE")
$\widehat{\beta} = 1.20,\ \widehat{\lambda} = 0.0538.$



Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours with Power and Loglinear NHPP Models for a USS Grampus Diesel Engine



Results of Fitting NHPP Models to the USS Grampus Diesel Engine Data

- Both models seem to fit the data very well.

- For the power recurrence rate model, $\widehat{\beta}$=1.22 and $\widehat{\eta}$ =0.553.

- For the loglinear recurrence rate model, $\widehat{\gamma}_0$=1.01 and $\widehat{\gamma}_1$ =.0377.

- Times between recurrences are consistent with a HPP:
  - the Lewis-Robinson test gave $Z_{\text{LR}} = 1.02$ with p-value $p = .21$.
  - the MIL-HDBk-189 test gave $X^2_{\text{MHB}} = 92$ with p-value $p = .08$.

Comparison of trend tests

main topic

Minitab provides five trend tests for data with multiple systems: MIL-hdbk-189 (TTT-based), MIL-hdbk-189 (Pooled), Laplace's (TTT-based), Laplace's (Pooled), and Anderson-Darling. The pooled Laplace and military handbook tests reduce to their respective TTT-based tests when there is only one system. These tests behave differently under the following two circumstances:

1 the data follow a non-monotonic trend

2 the data are from heterogeneous systems

Monotonic and non-monotonic trends

There is a trend in the pattern of times between failure if the times change in a systematic way. Trends can be:

- monotonic – times between failures are getting either consistently longer (decreasing trend) or consistently shorter (increasing trend)
- non-monotonic – times between failures alternate between increasing and decreasing trend (cyclic) or have a decreasing trend, no trend, and then increasing trend (bathtub)

The Anderson-Darling test will reject the null hypothesis in the presence of both monotonic and non-monotonic trends. The other tests will generally only detect monotonic trends. While the Anderson-Darling test is useful if you suspect the existence of a cyclic or other non-monotonic trend, the other tests are more powerful in the case of a monotonic trend.

Homogeneous and heterogeneous systems

The null hypothesis of no trend differs slightly for the different tests:

- The null hypothesis for the pooled tests (MIL-hdbk-189 and Laplace's) is that the data come from a homogeneous Poisson processes (HPP) with a possibly different MTBF for each system. Thus, rejecting the null hypothesis means that you can definitely conclude there is a trend in your data.
- The null hypothesis for the TTT-based tests (MIL-hdbk-189, Laplace's, and Anderson-Darling) is that the data come from a homogeneous Posson process (HPP) with the same MTBF for each system. Thus, rejecting the null hypothesis could mean that either there is a trend in your data or your data come from heterogeneous systems. Therefore, you should use TTT-based tests only when you are confident that your systems are homogeneous.

The table below summarizes the different null hypotheses associated with the trend tests.

	MIL-hdbk-189 (Pooled)	**MIL-hdbk-189 (TTT-based)**	**Laplace's (Pooled)**	**Laplace's (TTT-based)**	**Anderson-Darling**
Null Hypothesis	HPP (possibly different MTBFs)	HPP (equal MTBFs)	HPP (possibly different MTBFs)	HPP (equal MTBFs)	HPP (possibly different MTBFs)
Rejecting H_0 means...	monotonic trend	monotonic trend or systems are heterogeneous	monotonic trend	monotonic trend or systems are heterogeneous	monotonic trend or non-monotonic trend or systems are heterogeneous

See [12] for more information concerning these tests.

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TTT-based tests for trend in repairable systems data

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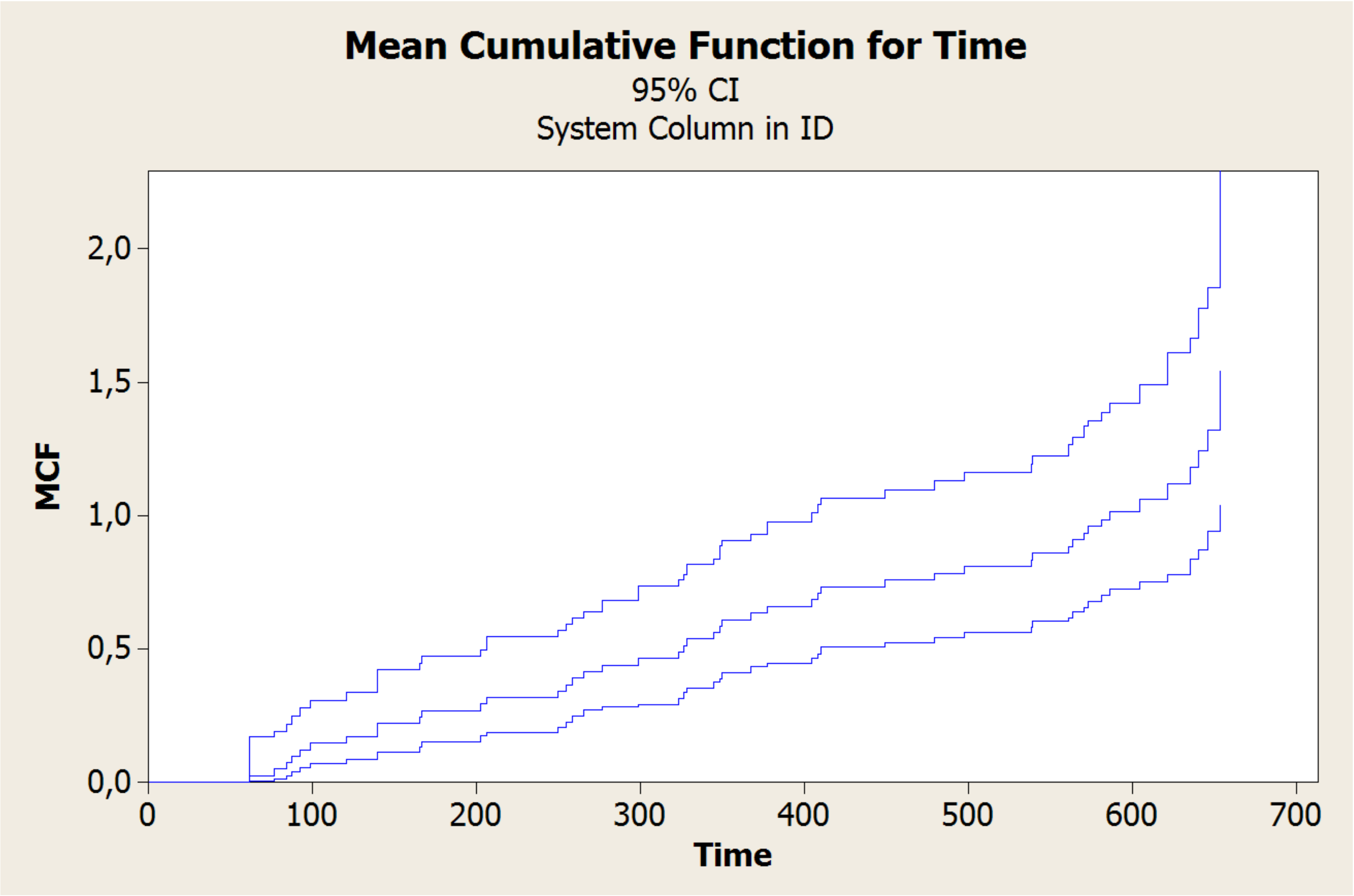
A major aspect of analysis of failure data for repairable systems is the testing for a possible trend in interfailure times. This paper reviews some important and popular graphical methods and tests for the nonhomogeneous Poisson process model. In particular, the total time on test (TTT) plot is considered, and trend tests based on the TTT-statistic are motivated and derived. In particular, a test based on the Anderson–Darling statistic is suggested. The tests are evaluated and compared in a simulation study, both with respect to the achievement of correct significance level and rejection power. The considered alternatives to 'no trend' are the log-linear, power law and a class of bathtub-shaped intensity functions. The simulation study involves single systems, as well as the case where several independent systems of the same kind are observed. © 1998 Elsevier Science Limited.

Valveseat Data



Valveseat Data

Trend Tests

	MIL-Hdbk-189		Laplace's		
	TTT-based	Pooled	TTT-based	Pooled	Anderson-Darling
Test Statistic	80,28	66,15	0,46	2,38	0,80
P-Value	0,249	0,017	0,645	0,017	0,478
DF	96	96			

TTT-analysis Simple Example

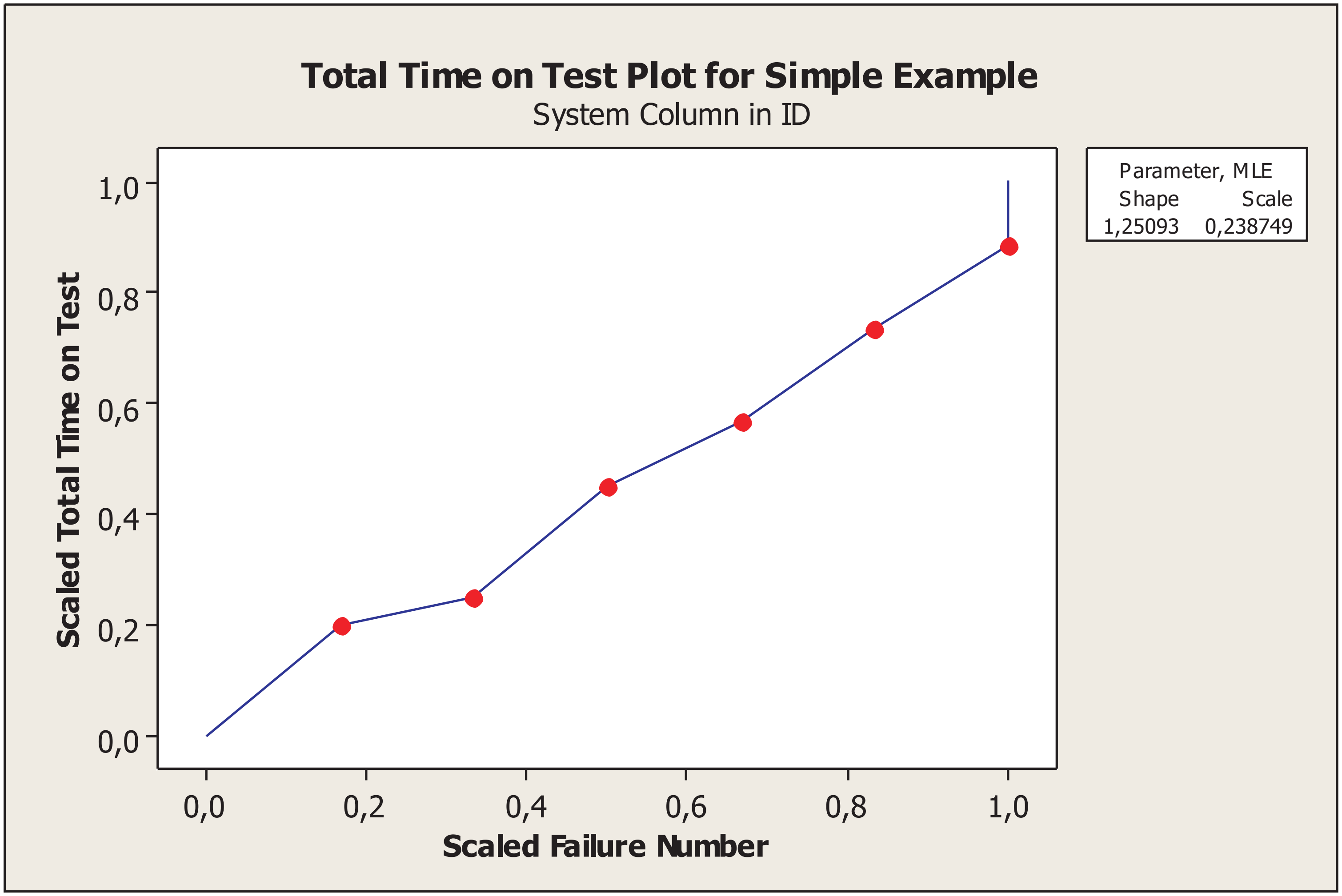
Row	STTT	ID	Scaled
1	12	1	0,20000
2	15	1	0,25000
3	27	1	0,45000
4	34	1	0,56667
5	44	1	0,73333
6	53	1	0,88333
7	60	1	1,00000

Parameter Estimates

		Standard	95% Normal CI	
Parameter	Estimate	Error	Lower	Upper
Shape	1,25093	0,511	0,249996	2,25186
Scale	0,238749	0,160	-0,0746105	0,552109

Trend Tests

	MIL-Hdbk-189	Laplace's	Anderson-Darling
Test Statistic	9,59	0,12	0,24
P-Value	0,697	0,906	0,977
DF	12		


Total Time on Test Plot for Simple Example
System Column in ID
Parameter, MLE
Shape Scale
1,25093 0,238749
Scaled Total Time on Test
1,0
0,8
0,6
0,4
0,2
0,0
0,0
0,2
0,4
0,6
0,8
1,0
Scaled Failure Number

TTT-analysis of Valve Seat Data

Parametric Growth Curve: C1

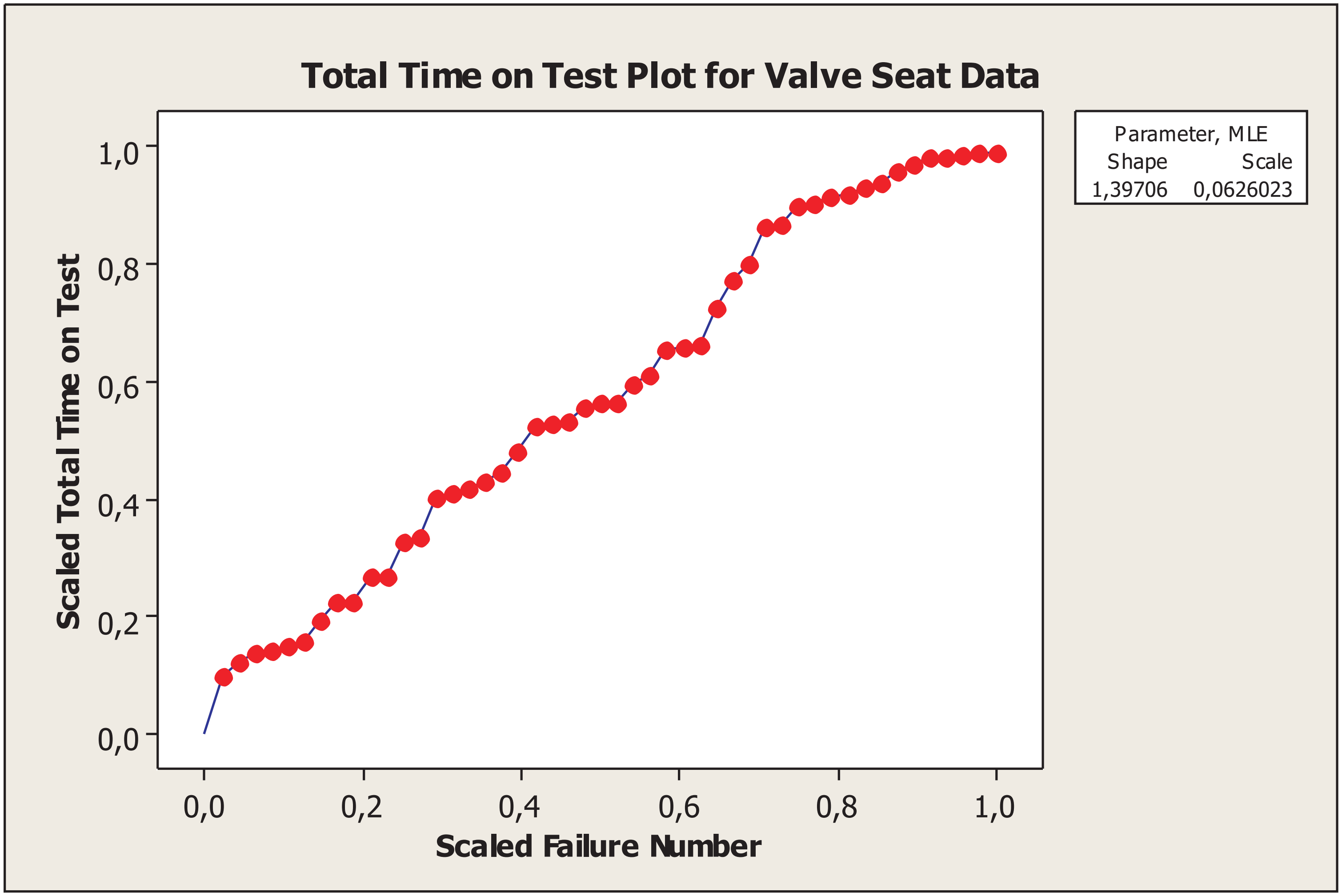
Model: Power-Law Process
Estimation Method: Maximum Likelihood

Parameter Estimates

		Standard	95% Normal CI	
Parameter	Estimate	Error	Lower	Upper
Shape	1,39706	0,202	1,00184	1,79229
Scale	0,0626023	0,026	0,0119179	0,113287

Trend Tests

	MIL-Hdbk-189	Laplace's	Anderson-Darling
Test Statistic	68,72	2,03	3,17
P-Value	0,032	0,043	0,022
DF	96		


Total Time on Test Plot for Valve Seat Data
Parameter, MLE
Shape Scale
1,39706 0,0626023
Scaled Total Time on Test
1,0
0,8
0,6
0,4
0,2
0,0
0,0
0,2
0,4
0,6
0,8
1,0
Scaled Failure Number