Imperfect repair models
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Single failure time T:
"Ordinary" hazard rate,

$$z(t) = \lim_{\Delta t \downarrow 0} \frac{Pr(T \in [t, t + \Delta t) | T > t)}{\Delta t}$$

Failure process $T_1, T_2, \ldots$:
CONDITIONAL INTENSITY, given history of failure process,

$$\phi(t|\mathcal{F}_{t-}) = \lim_{\Delta t \downarrow 0} \frac{Pr(\text{failure in } [t, t + \Delta t) | \mathcal{F}_{t-})}{\Delta t}$$

MINIMAL AND PERFECT REPAIR

Given hazard rate $z(t)$ of a *new* system

- PERFECT REPAIR $\Longleftrightarrow$ Renewal Process:

$$\phi(t|\mathcal{F}_{t-}) = z(t - T_{N(t)})$$

  System is as good as new after repair

- MINIMAL REPAIR $\Longleftrightarrow$ NHPP:

$$\phi(t|\mathcal{F}_{t-}) = z(t)$$

  Process continues as if no failures have ever occurred

IMPERFECT REPAIR – CLASSICAL MODEL

Brown & Proschan (1983):
A failure is

- perfectly repaired with probability p
- minimally repaired with probability $1 - p$

General idea: Effective age ('virtual age') models

$$\phi(t|\mathcal{F}_{t-}) = z\left(a_{N(t)} + t - T_{N(t)}\right)$$

where a_j is the *effective age* of system immediately after repair no. j

SPECIAL CASES:

- Perfect repair: $a_j = 0$ for all j
- Minimal repair: $a_j = T_j$
- Brown and Proschan (1983):

$$a_j = D_j(a_{j-1} + X_j)$$

where the D_j are independent with

$$D_j = \begin{cases} 1 \text{ with probability } 1-p & (\textit{minimal repair}) \\ 0 \text{ with probability } p & (\textit{perfect repair}) \end{cases}$$

Effective age models – more examples

Recall

$$\phi(t|\mathcal{F}_{t-}) = z\left(a_{N(t)} + t - T_{N(t)}\right)$$

where a_j is the *effective age* of system immediately after repair no. j, and put

$$a_j = D_j(a_{j-1} + X_j)$$

which is D_j times the effective age after last failure plus time elapsed since last failure

- Bathe and Franz (1996): The D_j are $1, c, 0$ with probabilities $p, q, 1 - p - q$.
- Kijima (1992), Model II: D_j general distribution on interval $[0, 1]$
- Kijima (1992), Model I:

$$a_j = a_{j-1} + D_j X_j$$

GENERALIZATIONS OF BROWN AND PROSCHAN'S MODEL

- Block et al. (1985): Extend Brown and Proschan's model to age-dependent repair, $p = p(t)$
- Iyer (1992): Extend age-dependent repair model to allow nonzero repair times
- Shaked & Shantikumar (1986): Imperfect repair model with several failure modes.

GENERALIZED LINEAR MODEL – Berman and Turner (1992):

$$\phi(t|\mathcal{F}_{t-}) = g\left\{\sum_{k=1}^{p} \theta_k Q_k(t|\mathcal{F}_{t-})\right\}$$

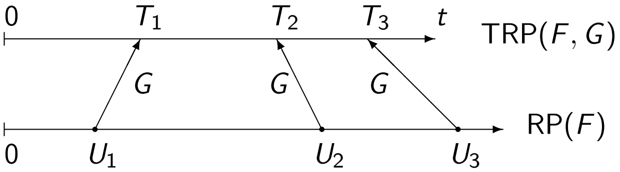
Example:

- Lawless and Thiagarajah (1996):

$$\begin{aligned} g(\cdot) &= \exp(\cdot) \\ Q_1(t|\mathcal{F}_{t-}) &= g_1(t) \\ Q_2(t|\mathcal{F}_{t-}) &= g_2(t - T_{N(t)}) \end{aligned}$$

TREND-RENEWAL PROCESS

- $U_1, U_2, \ldots$ is a renewal process
- $T_i = G(U_i)$ for G increasing function



SPECIAL CASES:

- NHPP: Renewal process is a homogeneous Poisson process
- RP: $G(u) = u$
- INHOMOGENEOUS GAMMA PROCESS – Berman (1981): Special case of TRP with F being a Gamma distribution, G arbitrary
- MODULATED POWER LAW PROCESS – Lakey and Rigdon (1992): Special case of TRP with F being a Gamma distribution and $G(u) = au^b$.