Inference in trend-renewal processes
Heterogeneity ("frailty") in repairable systems
STK4400 – Spring 2011

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Table 2. The Compressor Data

1	4	305	330	651	856	996	1,016	1,155	1,520	1,597	1,729
1,758	1,852	2,070	2,073	2,093	2,213	3,197	3,555	3,558	3,724	3,768	4,103
4,124	4,170	4,270	4,336	4,416	4,492	4,534	4,758	4,762	5,474	5,573	5,577
5,715	6,424	6,692	6,830	6,999	(7,571)						

NOTE: There are $n = 41$ failure times (in days), with time censoring at 7,571 days.

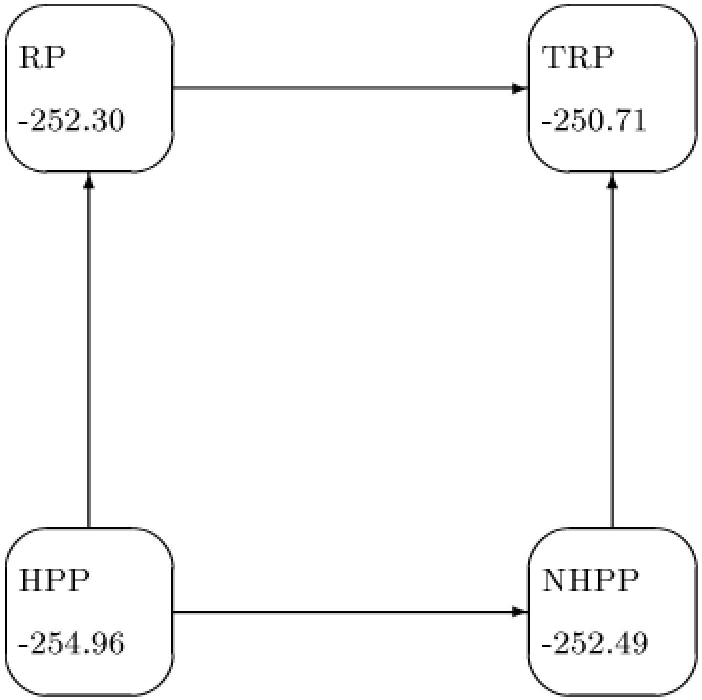
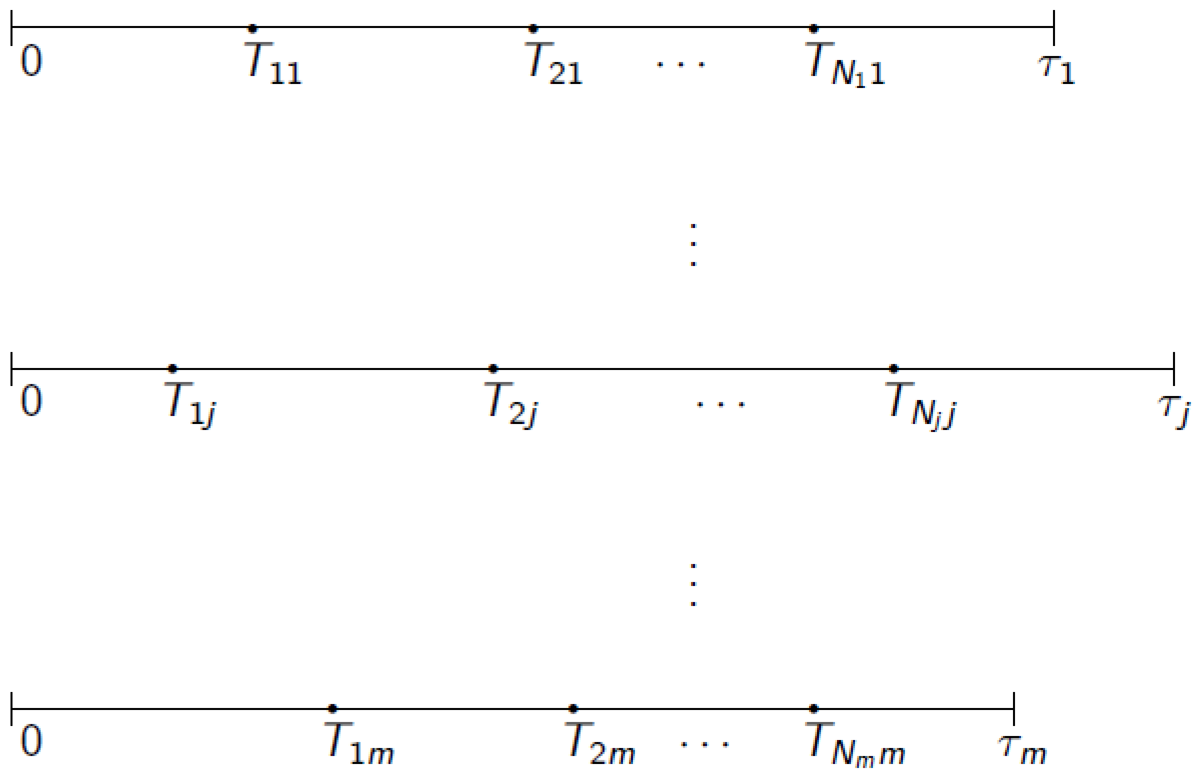


Figure 8. The Log-Likelihood Square of the TRP(F_w, $\lambda_{pl}(\cdot)$) Model for the Compressor Data.

Heterogeneity



Heterogeneity ("frailty") of systems

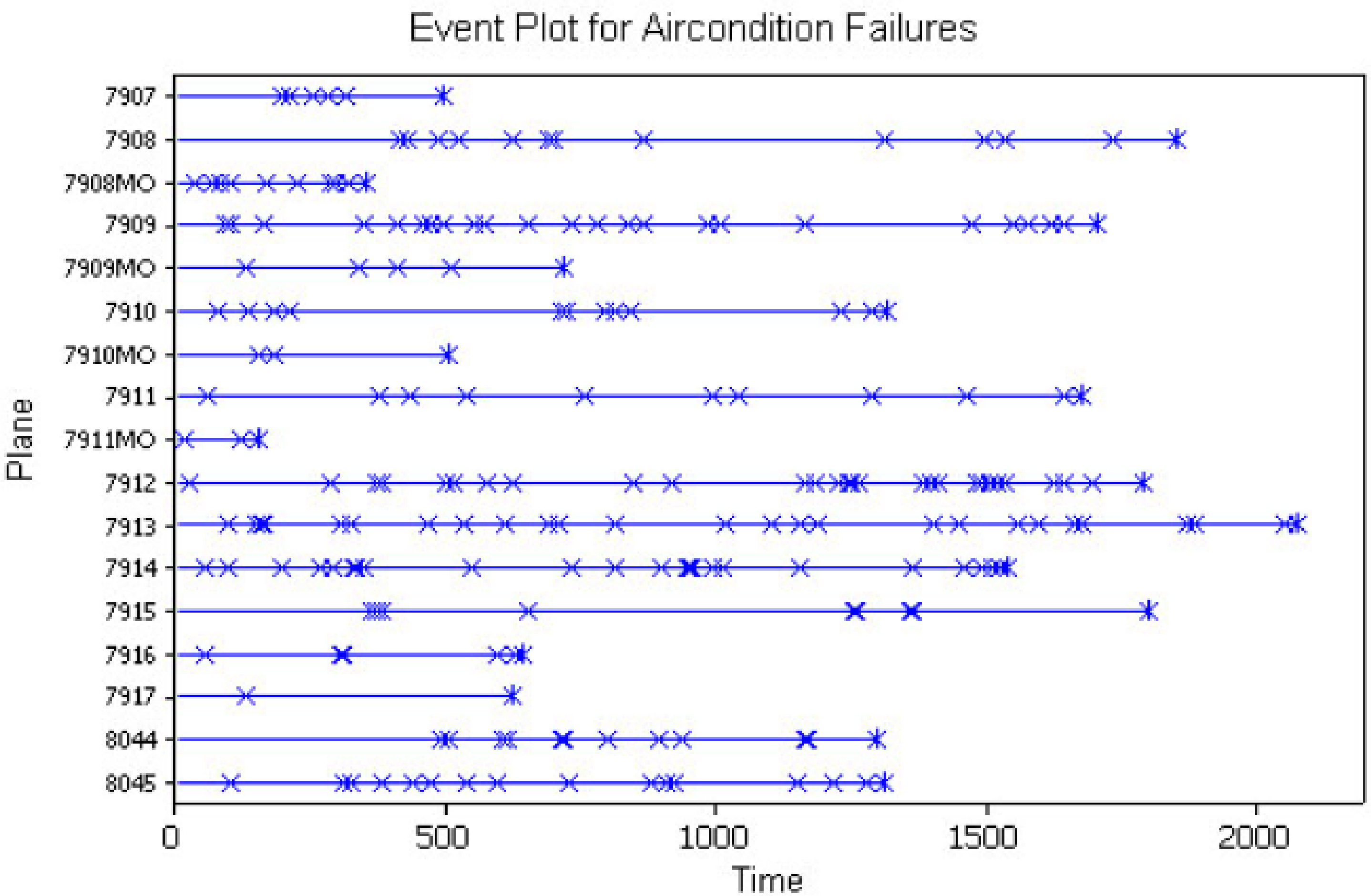
- Systems of the same kind may exhibit different failure intensities because of external unobserved sources:
  - Differing environmental conditions
  - Differing maintenance philosophies
  - Differing quality of operators
  - Differing quality of equipment
- These effects are modelled as unobserved random variables, commonly assumed to behave multiplicatively on a baseline intensity.

Example with NHPP:

- Failure intensity of jth system: $a_j \lambda(t)$
  where $a_1, \ldots, a_n$ are i.i.d. *unobservable* random variables from some positive distribution with expected value 1.

Proschan (1963): The classical "aircondition data"

Times of failures of aircondition system in a fleet of Boeing 720 airplanes



Proschan's analysis - heterogeneous Poisson processes

Proschan (1963):

"... it seems safe to accept the exponential distribution as describing the failure interval, although to each plane may correspond a different failure rate"

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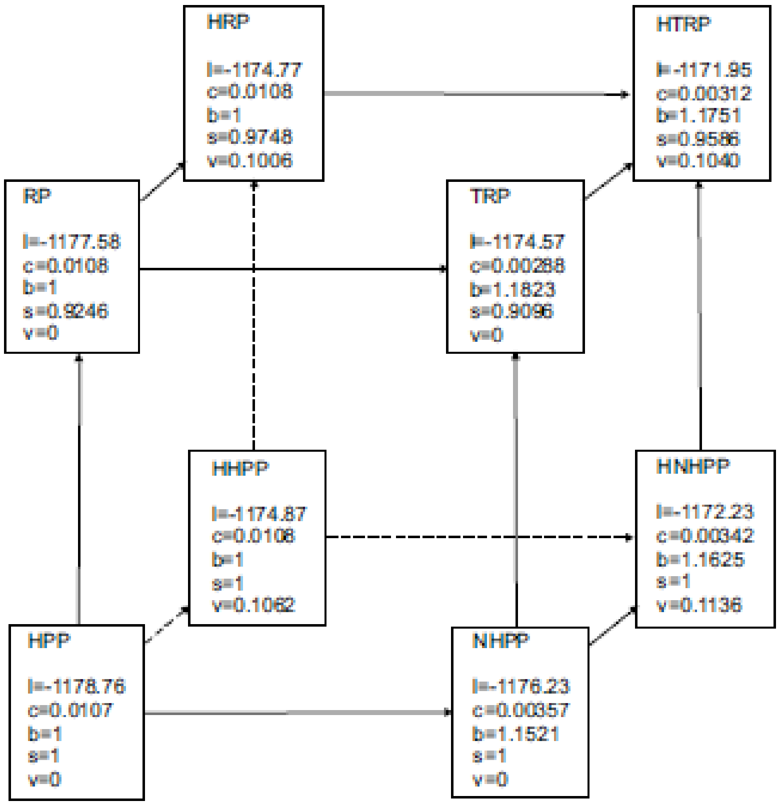
Analysis of Proschan's data



- F is Weibull-distribution with expected value 1 and shape parameter s
- $\lambda(t) = cbt^{b-1}$ is a power function of t
- H is gamma-distribution with expected value 1 and variance v.
- l is maximum value of the log likelihood.

Including unobserved heterogeneity in the TRP – the heterogeneous TRP (HTRP)

Definition of HTRP$(F, \lambda(\cdot), H)$

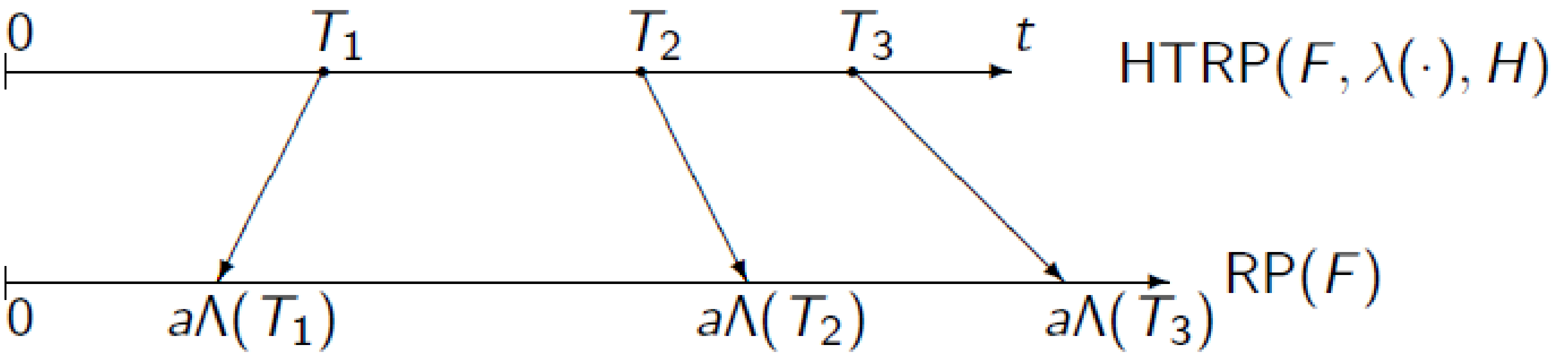
- m systems are observed
- jth system observed in $[0, \tau_j]$, with N_j observed failures

- Conditional on a_j is process j a TRP$(F, \lambda_j(\cdot))$ where

$$\lambda_j(t) = a_j \lambda(t)$$

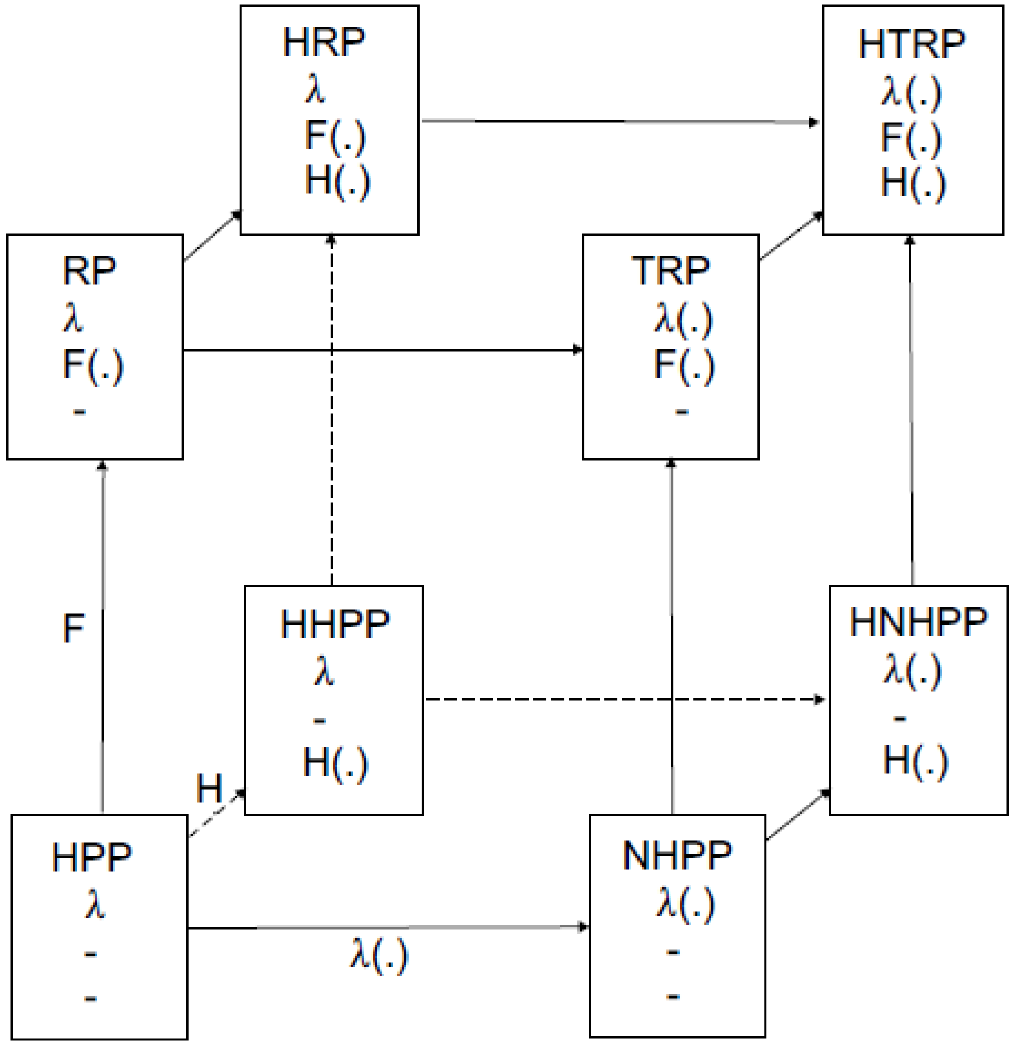
- The a_j are i.i.d. (unobserved) random variables with d.f. H, expected value 1.

The seven submodels of HTRP$(F, \lambda(\cdot), H)$



Submodel	*HTRP-formulation*
HPP(ν)	HTRP$(\exp, \nu, 1)$
RP(F, ν)	HTRP$(F, \nu, 1)$
NHPP$(\lambda(\cdot))$	HTRP$(\exp, \lambda(\cdot), 1)$
TRP$(F, \lambda(\cdot))$	HTRP$(F, \lambda(\cdot), 1)$
HHPP(ν, H)	HTRP$(\exp, \nu, H)$
HRP(F, ν, H)	HTRP(F, ν, H)
HNHPP$(\lambda(\cdot), H)$	HTRP$(\exp, \lambda(\cdot), H)$

HTRP and its submodels – the model cube



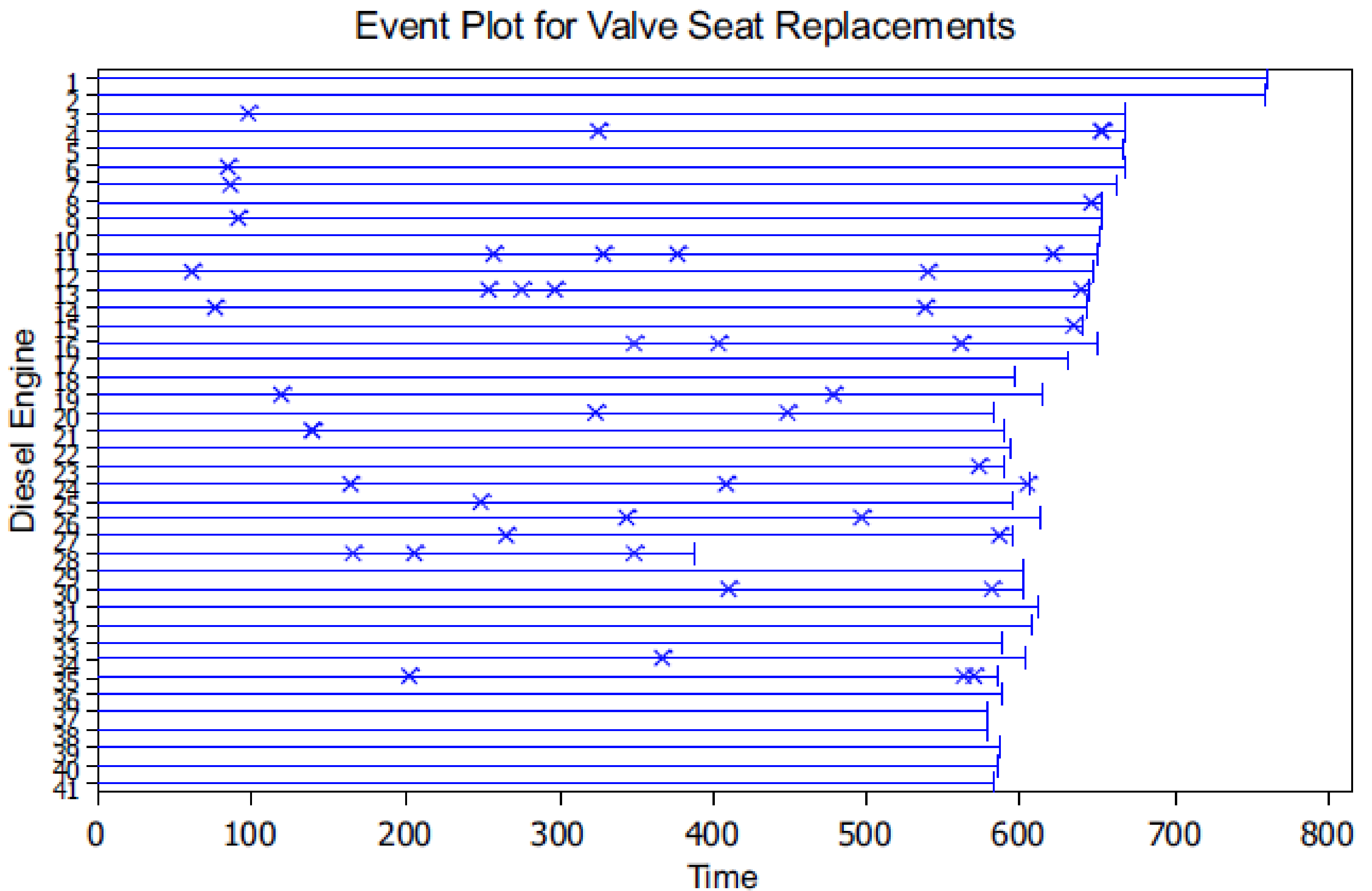
Parametric inference in HTRPs

Example model: HTRP$(F, \lambda(\cdot), H)$ and submodels

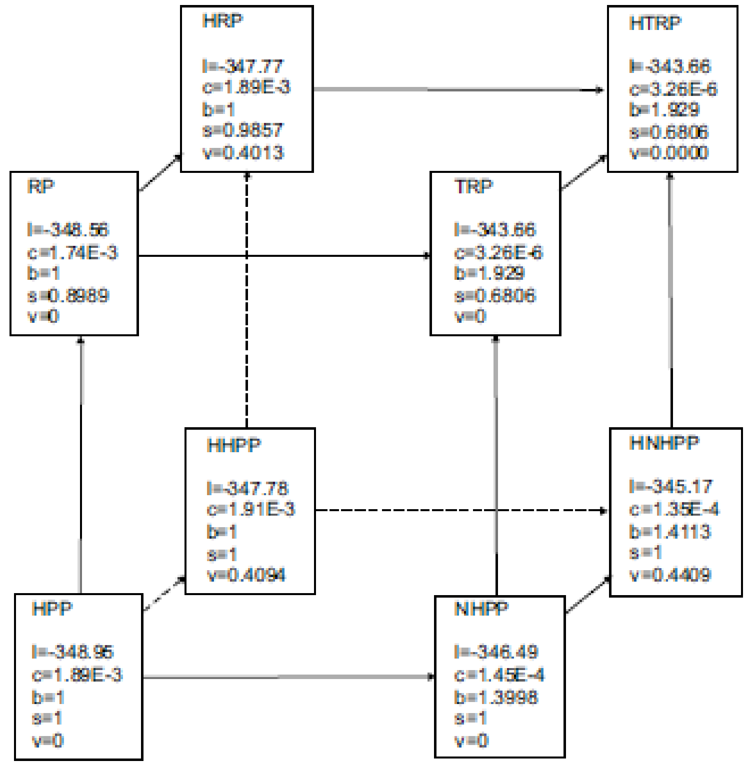
- F (renewal distribution):
  - Weibull distribution (expected value 1, shape parameter s)
  - *Submodel: Standard exponential distribution*
- $\lambda(\cdot)$ (trend function): Power law,
  - $\lambda(t) = cbt^{b-1}$
  - *Submodel* : $\lambda(t) = c$
- H (heterogeneity distribution):
  - Gamma distribution (expected value 1)
  - *Submodel:* H is degenerate at 1

Nelson (1995): Valve seat data

Times of valve-seat replacements in a fleet of 41 diesel engines



Analysis of Nelson's data



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