Sequential optimization of oil production from multiple reservoirs under uncertainty

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Abstract: In the present paper we study how to optimize oil production from several reservoirs sharing a common processing facility under uncertainty. The potential oil production from a reservoir in a given period is described in terms of a difference equation containing several uncertain parameters. Using Lagrange optimization a step by step production strategy can be determined. This strategy distributes the available processing capacity so that the expected total production from each period is maximized. This is done by ensuring that all the reservoirs have the same probability of producing according to the plan. The resulting strategy, however, is typically not optimal globally. In order to improve the global performance of the strategy, a different objective function is introduced. As the production develops, more information about the production parameters is gained. Hence, the uncertainty distributions need to be updated. This is done using a combination of rejection sampling and the Metropolis-Hastings algorithm. This updating is taken into account in the optimization procedure. The methods are illustrated by considering a specific example.

Keywords: Sequential Stochastic Optimization, Oil Production, Monte Carlo Simulation.

1. INTRODUCTION

Optimization is an important element in the management of multiple-field oil and gas assets, since many investment decisions are irreversible and finance is committed for the long term. Recent studies of production optimization include [1], [2] and [3]. [4] considered the problem of production optimization in an oil or gas field consisting of many reservoirs sharing the same processing facility. In order to satisfy the processing limitations of the facility, the production needs to be choked. Thus, at any given point of time the production from each of the reservoirs are scaled down by suitable *choke factors* chosen so that the total production does not exceed the processing capacity. Based on the production profile models introduced in [5], [6] developed a general model for oil and gas production using a set of ordinary differential equations. [4] used this approach in order to develop a general framework for finding a production strategy which is optimal with respect to various types of objective functions. In [7] this work was extended to cases where the production is uncertain. In [8] a discrete time version of the same problem was discussed, focussing on a case where the production from a single reservoir was optimized relative to a certain cost and income function. In the present paper we extend this to cases with multiple reservoirs.

In each time period the production is limited by two factors: the potential *production capacities* for each of the reservoirs, and the total *processing capacity*, i.e., the total amount of oil that can be handled at the processing facility. In order to limit the amount of oil that is produced from the reservoirs during a given time period, *processing quotas* are assigned to each of the reservoirs at the beginning of each time period. The sum of these quotas are not allowed to exceed the total processing capacity. However, if the quota assigned to a reservoir is greater than its potential production capacity, the quota cannot be fully utilized in that time period. As a result the sum of the produced volumes for that period may be less than the total processing capacity.

Considering a single time period the quotas for this period should be assigned so that the expected total production from this period is maximized. If the potential production capacities for the reservoirs are known, it is easy to ensure this. However, the potential production capacities typically depends on a number of uncertain reservoir parameters. Thus, in order to choose the optimal quotas, one has to take this uncertainty into account.

The uncertainty about the reservoir parameters is expressed in terms of a suitable prior distribution. As the production develops, more information about the production parameters is gained. Hence, the uncertainty distributions need to be updated. In the present paper we show how this updating can be accomplished. Moreover, we show how to assign quotas so that the expected total production for a given time period is maximized.

Unfortunately, assigning quotas maximizing the expected production for each time period may not necessarily be optimal when all the periods are considered simultaneously. In the paper we indicate how to establish reasonable strategies for handling this issue.

2. PROBLEM DESCRIPTION

We consider n oil reservoirs, and assume that the production from each reservoir is described as a discrete time process, where q_{ik} denotes the production from the ith reservoir in the kth period, $i = 1, \ldots, n$ and $k = 1, 2, \ldots$. We also introduce the *cumulative* production from the ith reservoir adding up the production from the periods $1, \ldots, k$, denoted Q_{ik}, $k = 1, 2, \ldots$. That is, for $i = 1, \ldots, n$ we let:

$$Q_{ik} = \sum_{j=1}^{k} q_{ij} \quad k = 1, 2, \ldots \tag{1}$$

For convenience, we also define $Q_{i0} = 0$, $i = 1, \ldots, n$.

The amount of oil that can be produced from a reservoir within a period is limited by the characteristics of the reservoir determined by a set of uncertain parameters. The main reservoir parameters are the *amount of recoverable oil*, denoted by V_i, and the so-called *decline rate*, denoted by D_i, $i = 1, \ldots, n$. The decline rate represents the fraction of remaining oil that can be produced per unit of time. In general the decline rate may change over time. Still using a production model with a constant decline rate may serve as a satisfactory approximation in many situations. The maximum amount of oil that can be produced from the ith reservoir within the kth period given no other restrictions, can then for $i = 1, \ldots, n$ and $k = 1, 2, \ldots$ be expressed as:

$$f_i(Q_{i,k-1}) = D_i(V_i - Q_{i,k-1}). \tag{2}$$

We refer to the function f_i as the potential production rate function, or PPR function of the ith reservoir.

Obviously, the amount of recoverable oil is some non-negative number, while the decline rate must be a number between zero and one. Thus, for $i = 1, \ldots, n$ the set of possible values for the vector (V_i, D_i) is given as:

$$\mathcal{X}_i = \mathbb{R}_+ \times [0, 1]. \tag{3}$$

Moreover, we assume that the distribution of (V_i, D_i) has a prior density $\pi_i(v, d)$ defined over the set $\mathcal{X}_i$, $i = 1, \ldots, n$, and that the vectors $(V_1, D_1), \ldots (V_n, V_n)$ are independent of each other.

In addition to the restrictions imposed by the reservoir itself, the actual production is typically restricted by the available processing capacity. Here we consider the case where the processing facilities are shared between the n reservoirs and where K denotes the total processing capacity at these facilities. As a result the reservoir manager needs to assign processing quotas for each reservoir at the start of each period. Thus, we let x_{ik} denote the processing quota assigned to the ith reservoir during the kth production period, $i = 1, 2, \ldots, n$, $k = 1, 2, \ldots$. Moreover, we let $\boldsymbol{x}_k = (x_{1k}, \ldots, x_{nk})$, $k = 1, 2, \ldots$. Hence, for $i = 1, \ldots, n$ and $k = 1, 2, \ldots$ the actual production volumes are given by:

$$q_{ik} = q_{ik}(x_{ik}) = \min\{f_i(Q_{i,k-1}), x_{ik}\}. \tag{4}$$

We assume that the quotas are chosen so that the following condition is satisfied:

$$\sum_{i=1}^{n} x_{ik} = K, \quad k = 1, 2, \ldots. \tag{5}$$

We now consider the kth time period, and note that if $x_{ik} \leq f_i(Q_{i,k-1})$ for $i = 1, \ldots, n$, it follows by (4) that $q_{ik} = x_{ik}$, $i = 1, \ldots, n$. Thus, in this case all the quotas are fully utilized, and by (5) we get that:

$$\sum_{i=1}^{n} q_{ik} = K.$$

If on the other hand $x_{jk} > f_j(Q_{j,k-1})$ for some j, the quota for this reservoir is *not* fully utilized. Hence, in this case:

$$\sum_{i=1}^{n} q_{ik} < K.$$

That is, some of the available processing capacity is not utilized. Obviously, a good production strategy should aim at minizing the probability of this happening. Still if the reservoir parameters are uncertain quantities, this will surely happen from time to time.

The main focus in this paper is on cases where the reservoir parameters are uncertain quantities. However, we close this section by considering briefly how to optimize production in a case where the reservoir parameters are known. In [4] this problem was investigated in a continuous time setting. When a continuous time model is used, the difference equations for the PPR-functions is replaced by differential equations. [4] showed that when all the PPR-functions satisfy linear differential equations, the optimal production strategy is to use a *strict priority rule*, where the reservoirs are prioritized according to their respective decline rates. In particular the reservoir with the lowest decline rate should be given the highest priority, while the reservoir with the highest decline rate should be given the lowest priority. It turns out that this result is valid in the discrete time case as well. This result will be included in an upcoming paper.

3. SHORT-TERM OPTIMIZATION IN THE STOCHASTIC CASE

In this section we present a possible solution to production optimization under uncertainty. A reasonable approach to this problem is to assign quotas so that the expected total production is maximized. In order to do so we introduce:

$$Y_k = Y_k(\boldsymbol{x}_k) = \sum_{i=1}^{n} q_{ik}, \quad k = 1, 2, \ldots. \tag{6}$$

Considering the kth time period, the objective is then to choose $\boldsymbol{x}_k$ so that $E[Y_k(\boldsymbol{x}_k)]$ is maximized subject to the constraint (5). Note that by using this approach at each step, we focus on the upcoming time period only. Thus, the solution may not be optimal in a long-term perspective. We refer to this optimization problem as the *short-term optimization problem*, and the solution will be called the *short-term production strategy*.

In order to solve the short-term optimization problem, we introduce the Lagrange function:

$$\Lambda_S(\boldsymbol{x}_k, \lambda) = \Phi_S(\boldsymbol{x}_k) - \lambda \Psi(\boldsymbol{x}_k), \tag{7}$$

where λ denotes the Lagrange multiplier, and where $\Phi_S(\boldsymbol{x}_k) = E[Y_k(\boldsymbol{x}_k)]$ and $\Psi(\boldsymbol{x}_k) = \sum_{i=1}^{n} x_{ik} - K$. A stationary point for the Lagrange function is then found by solving the equation:

$$\nabla \Phi_S(\boldsymbol{x}_k) = \lambda \nabla \Psi(\boldsymbol{x}_k), \tag{8}$$

subject to the restriction that $\Psi(\boldsymbol{x}_k) = 0$. It is easy to verify that Φ_S is a *concave* function. Thus, the stationary point will be a maximum point, which implies that this will indeed be a solution to our optimization problem. Furthermore, under mild restrictions the derivative of the expectation is equal to the expected value of the derivative. Thus, for $i = 1, \ldots, n$ we get:

$$\begin{aligned}
\frac{\partial}{\partial x_{ik}} \Phi_S(\boldsymbol{x}_k) &= \frac{\partial}{\partial x_{ik}} E[Y_k(\boldsymbol{x}_k)] \\
&= E[\frac{\partial}{\partial x_{ik}} Y_k(\boldsymbol{x}_k)] \\
&= E[\frac{\partial}{\partial x_{ik}} \min\{f_i(Q_{i,k-1}), x_{ik}\}] \\
&= E[I(f_i(Q_{i,k-1}) > x_{ik})] \\
&= P(f_i(Q_{i,k-1}) > x_{ik}).
\end{aligned} \tag{9}$$

where $I(\cdot)$ denotes the indicator function. Moreover, we get:

$$\nabla \Psi(\boldsymbol{x}_k) = (1, \ldots, 1). \tag{10}$$

By inserting (9) and (10) into (8), we get that the optimal solution must satisfy:

$$P(f_i(Q_{i,k-1}) > x_{ik}) = \lambda, \quad i = 1, \ldots n, \tag{11}$$

for some value of λ, as well as the restriction (5). In order to find a solution to these equations, we introduce the non-increasing functions $\bar{F}_{ik}(x_{ik}) = P(f_i(Q_{i,k-1}) > x_{ik})$, $i = 1, \ldots, n$ and $k = 1, 2, \ldots$. Noting that the $\bar{F}_{ik}$-functions all have values ranging over the interval $[0, 1]$, we also introduce:

$$\begin{aligned}
x_{ik}^L &= \sup\{x : \bar{F}_{ik}(x_{ik}) = 1\}, \quad i = 1, \ldots, n, \quad k = 1, 2, \ldots, \\
x_{ik}^H &= \inf\{x : \bar{F}_{ik}(x_{ik}) = 0\}, \quad i = 1, \ldots, n, \quad k = 1, 2, \ldots.
\end{aligned}$$

Note that since potential production rates obviously are non-negative numbers, we must have:

$$0 \leq x_{ik}^L \leq x_{ik}^H \leq \infty, \quad i = 1, \ldots, n, \quad k = 1, 2, \ldots.$$

In the following discussion we assume that $\bar{F}_{ik}$ is continuous and strictly decreasing[1] on the interval $[x_{ik}^L, x_{ik}^H]$, $i = 1, \ldots, n$, $k = 1, 2, \ldots$. We may then define an inverse mapping:

$$\bar{F}_{ik}^{-1} : [0, 1] \to [x_{ik}^L, x_{ik}^H], \quad i = 1, \ldots, n, \quad k = 1, 2, \ldots.$$

Using this inverse mapping we may express $x_{1k}, \ldots, x_{nk}$ in terms of λ and insert the results into (5):

$$\sum_{i=1}^{n} \bar{F}_{ik}^{-1}(\lambda) = K, \quad k = 1, 2, \ldots. \tag{12}$$

The task is thus reduced to solving (12) with respect to the single remaining unknown λ. As soon as λ is found, the resulting quotas $x_{1k}, \ldots, x_{nk}$ are determined by using the inverse mapping. In order to solve (12) we consider three different cases:

Case 1. $\sum_{i=1}^{n} x_{ik}^H < K$. In this case (12) does not have a solution. Still it is possible to find a solution to the original set of equations (11) subject to the restriction (5) simply by letting:

$$x_{ik} = \frac{K \cdot x_{ik}^H}{\sum_{j=1}^{n} x_{jk}^H}, \quad i = 1, \ldots, n. \tag{13}$$

It is easy to verify that (13) indeed satisfies the restriction (5). Moreover, since $x_{ik} \geq x_{ik}^H$, $i = 1, \ldots, n$, and since the $\bar{F}_{ik}$-functions are non-increasing, it follows that $\bar{F}_{ik}(x_{ik}) = \bar{F}_{ik}(x_{ik}^H) = 0$, $i = 1, \ldots n$.

[1] If this is not the case, the calculations need to be modified slightly, but we skip these details here.

CASE 2. $\sum_{i=1}^{n} x_{ik}^{L} > K$. As in the previous case (12) does not have a solution. In this case a solution to the original set of equations (11) subject to the restriction (5) can be found by letting:

$$x_{ik} = \frac{K \cdot x_{ik}^{L}}{\sum_{j=1}^{n} x_{jk}^{L}}, \quad i = 1, \ldots, n. \tag{14}$$

Again it is easy to verify that (14) satisfies the restriction (5). Moreover, since $x_{ik} \leq x_{ik}^{L}$, $i = 1, \ldots, n$, and since the $\bar{F}_{ik}$-functions are non-increasing, it follows that $\bar{F}_{ik}(x_{ik}) = \bar{F}_{ik}(x_{ik}^{L}) = 1$, $i = 1, \ldots n$.

CASE 3. $\sum_{i=1}^{n} x_{ik}^{L} \leq K \leq \sum_{i=1}^{n} x_{ik}^{H}$. In this case it follows by the assumption that the $\bar{F}_{ik}$'s are continuous and strictly decreasing that (12) has a unique solution. The solution can be found either analytically, or by a simple numerical method like e.g., the well-known *bisection method*. In particular since the set of possible λ-values is limited to the set $[0, 1]$, finding the solution with a sufficient precision is easy.

4. LONG-TERM OPTIMIZATION IN THE STOCHASTIC CASE

We start this section by noting that the short-term production strategy found in the previous attempts to balance the risk of not being able to utilize the assigned quotas between the n reservoirs. Thus, compared to the optimal strict priority strategy for the deterministic case the short-term strategy behaves quite differently. Motivated by this we now introduce a different approach for the stochastic case where the long-term perspective is given more weight.

In the deterministic case the optimal strategy is to make sure that the reservoirs with lowest decline rates are produced first. As a result, the tail-production will be dominated by the reservoirs with the highest decline rates. Thus, the remaining volumes will be produced as fast as possible.

One way of evaluating the tail-production is by calculating its *potential center of mass*. In order to define this concept, we start out by considering the ith reservoir, and assume that we have completed $k-1$ periods of production. Using the notation introduced in the previous section, the production in the kth period is q_{ik}. Assuming that the reservoir is produced at maximum speed in all the periods following the kth period, we get that:

$$\begin{aligned}
q_{i,k+1} &= D_i(V_i - Q_{i,k-1} - q_{ik}), \\
q_{i,k+2} &= D_i(V_i - Q_{i,k} - q_{i,k+1}) = D_i(1 - D_i)(V_i - Q_{i,k-1} - q_{ik}), \\
&\vdots \\
q_{i,k+h} &= D_i(V_i - Q_{i,k+h-2} - q_{i,k+h-1}) = D_i(1 - D_i)^{h-1}(V_i - Q_{i,k-1} - q_{ik}).
\end{aligned}$$

The potential center of mass for the tail-production after the kth period of the ith reservoir, expressed as a function of x_{ik}, and denoted by $Z_{ik}(x_{ik})$, is defined as:

$$\begin{aligned}
Z_{ik}(x_{ik}) &= \sum_{h=1}^{\infty} h \cdot q_{i,k+h} \\
&= \sum_{h=1}^{\infty} h \cdot D_i(1 - D_i)^{h-1}(V_i - Q_{i,k-1} - q_{ik}(x_{ik})) \\
&= (V_i - Q_{i,k-1} - q_{ik}(x_{ik})) \cdot D_i^{-1}.
\end{aligned}$$

Furthermore, the potential center of mass for the tail-production after the kth period of all reservoirs combined, expressed as a function of $\boldsymbol{x}_k$, and denoted by $Z_k(\boldsymbol{x}_k)$, is defined as:

$$\begin{aligned}
Z_k(\boldsymbol{x}_k) &= \sum_{i=1}^{n} Z_{ik}(x_{ik}) \\
&= \sum_{i=1}^{n} (V_i - Q_{i,k-1} - q_{ik}(x_{ik})) \cdot D_i^{-1}.
\end{aligned}$$

In order to optimize the tail-production, we seek a strategy such that expected potential center of mass is as low as possible, since this implies that the remaining volumes can be produced as fast as possible. We refer to this optimization problem as the *long-term optimization problem*, and the solution will be called the *long-term production strategy.*

In order to solve the long-term optimization problem, we again introduce the Lagrange function:

$$\Lambda_L(\boldsymbol{x}_k, \lambda) = \Phi_L(\boldsymbol{x}_k) + \lambda\Psi(\boldsymbol{x}_k), \tag{15}$$

where λ in this case conveniently denotes the *negative* Lagrange multiplier, and where $\Phi_L(\boldsymbol{x}_k) = E[Z_k(\boldsymbol{x}_k)]$ and $\Psi(\boldsymbol{x}_k) = \sum_{i=1}^n x_{ik} - K$. A stationary point for the Lagrange function is then found by solving the equation:

$$-\nabla\Phi_L(\boldsymbol{x}_k) = \lambda\nabla\Psi(\boldsymbol{x}_k), \tag{16}$$

subject to the restriction that $\Psi(\boldsymbol{x}_k) = 0$. It is easy to verify that Φ_L is a *convex* function. Thus, the stationary point will be a minimum point, which implies that this will indeed be a solution to our optimization problem. Under the same mild restrictions as in the short-term case the derivative of the expectation is equal to the expected value of the derivative. Thus, for $i = 1, \ldots, n$ we get:

$$\begin{aligned} -\frac{\partial}{\partial x_{ik}}\Phi_L(\boldsymbol{x}_k) &= -\frac{\partial}{\partial x_{ik}}E[Z_k(\boldsymbol{x}_k)] \\ &= -E[\frac{\partial}{\partial x_{ik}}Z_k(\boldsymbol{x}_k)] \\ &= E[\frac{\partial}{\partial x_{ik}}\min\{f_i(Q_{i,k-1}), x_{ik}\} \cdot D_i^{-1}] \\ &= E[I(f_i(Q_{i,k-1}) > x_{ik}) \cdot D_i^{-1}]. \end{aligned} \tag{17}$$

Moreover, $\nabla\Psi(\boldsymbol{x}_k)$ is, as before, given by (10). By inserting (17) and (10) into (16), we get that the optimal solution must satisfy:

$$E[I(f_i(Q_{i,k-1}) > x_{ik}) \cdot D_i^{-1}] = \lambda, \quad i = 1, \ldots n, \tag{18}$$

for some value of λ, as well as the restriction (5). In order to find a solution to these equations, we use the same procedure as we used for the short-term problem. Thus, we introduce the non-increasing functions $\bar{G}_{ik}(x_{ik}) = E[I(f_i(Q_{i,k-1}) > x_{ik}) \cdot D_i^{-1}]$, $i = 1, \ldots, n$ and $k = 1, 2, \ldots$. We observe that $\bar{G}_{ik}$ has values ranging over the interval $[0, \mu_i]$, where $\mu_i = E[D_i^{-1}]$, $i = 1, \ldots, n$ and $k = 1, 2, \ldots$. Since we are looking for solution vectors, $\boldsymbol{x}_k$ so that the $\bar{G}_{ik}$-functions become equal, we need to limit our search for λ-values to the interval $[0, \mu]$, where $\mu = \min\{\mu_1, \ldots, \mu_n\}$. Keeping this in mind, we introduce:

$$\begin{aligned} x_{ik}^L &= \sup\{x : \bar{G}_{ik}(x_{ik}) = \mu\}, \quad i = 1, \ldots, n, \quad k = 1, 2, \ldots, \\ x_{ik}^H &= \inf\{x : \bar{G}_{ik}(x_{ik}) = 0\}, \quad i = 1, \ldots, n, \quad k = 1, 2, \ldots. \end{aligned}$$

As in the short-term problem, we must have:

$$0 \leq x_{ik}^L \leq x_{ik}^H \leq \infty, \quad i = 1, \ldots, n, \quad k = 1, 2, \ldots.$$

Proceeding as we did previously, we assume that $\bar{G}_{ik}$ is continuous and strictly decreasing on the interval $[x_{ik}^L, x_{ik}^H]$, $i = 1, \ldots, n$, $k = 1, 2, \ldots$, and define an inverse mapping:

$$\bar{G}_{ik}^{-1} : [0, \mu] \to [x_{ik}^L, x_{ik}^H], \quad i = 1, \ldots, n, \quad k = 1, 2, \ldots.$$

Using this inverse mapping we express $x_{1k}, \ldots, x_{nk}$ in terms of λ and insert the results into (5):

$$\sum_{i=1}^n \bar{G}_{ik}^{-1}(\lambda) = K, \quad k = 1, 2, \ldots. \tag{19}$$

Thus, again we reduce the problem to solving (19) with respect to the single remaining unknown λ from which the resulting quotas $x_{1k}, \dots, x_{nk}$ are determined by using the inverse mapping. In order to solve (19) we again need to consider three different cases:

Case 1. $\sum_{i=1}^{n} x_{ik}^{H} < K$. In this case (19) does not have a solution. Still it is possible to find a solution to the original set of equations (18) subject to the restriction (5) simply by letting:

$$x_{ik} = \frac{K \cdot x_{ik}^{H}}{\sum_{j=1}^{n} x_{jk}^{H}}, \quad i = 1, \dots, n. \tag{20}$$

It is easy to verify that (20) indeed satisfies the restriction (5). Moreover, since $x_{ik} \geq x_{ik}^{H}$, $i = 1, \dots, n$, and since the $\bar{G}_{ik}$-functions are non-increasing, it follows that $\bar{G}_{ik}(x_{ik}) = \bar{G}_{ik}(x_{ik}^{H}) = 0$, $i = 1, \dots n$.

Case 2. $\sum_{i=1}^{n} x_{ik}^{L} > K$. As in the previous case (19) does not have a solution. However, contrary to the short-term problem, we cannot find a solution to the original set of equations simply by scaling down all the x_{ik}s, as this may alter the values of some of the $\bar{G}_{ik}$s. Instead we focus on the reservoir (or reservoirs) having the smallest μ_i-value, i.e., μ. Say e.g., that the index of this reservoir is j. For this particular reservoir we may safely scale down its quota x_{jk} without increasing the value of $\bar{G}_{jk}(x_{jk})$, since this value is already at its maximum value. Note that this reservoir most likely also has the highest decline rate, so it makes perfect sense to save the oil in this reservoir for later periods.

If $\sum_{i \neq j} x_{ik}^{L} \leq K$, it is possible to find a solution to the original set of equations (18) subject to the restriction (5) by choosing x_{jk} to be a sufficiently small non-negative number. If on the other hand $\sum_{i \neq j} x_{ik}^{L} > K$, there exists no solution to the original set of equations. In order to proceed, we temporarily eliminate the jth reservoir from the problem, and run the algorithm on the remaining reservoirs. That is, we replace μ by the smallest value of μ_i in the reduced set of reservoirs. Using this new μ-value, we recalculate $x_{1k}^{L}, \dots, x_{nk}^{L}$ (skipping x_{jk} since this is currently eliminated from the set). Since the new μ-value is greater than the old μ-value, the revised values of $x_{1k}^{L}, \dots, x_{nk}^{L}$ will typically be smaller than the previous values. If the sum of these values are smaller than or equal to K, the optimal quotas can be found using the method described in Case 3 below. If on the other hand the sum is still greater than K, we reduce the quota for the reservoir with the smallest μ-value. This process of gradually eliminating reservoirs continues until we find a combination of quotas whose sum is equal to K.

Case 3. $\sum_{i=1}^{n} x_{ik}^{L} \leq K \leq \sum_{i=1}^{n} x_{ik}^{H}$. By the assumption that the $\bar{G}_{ik}$'s are continuous and strictly decreasing it follows that (19) has a unique solution. As in the short-term problem the solution can be found either analytically, or by a simple numerical method like e.g., the well-known *bisection method.* In particular since the set of possible λ-values is limited to the set $[0, \mu]$, finding the solution with a sufficient precision is easy.

By assessing probability distributions for the reservoir parameters, we estimate the $\bar{F}_{ik}$-functions, $\bar{G}_{ik}$-functions as well as their respective inverses. When $k = 1$ it may sometimes be possible to derive these functions analytically. As the production develops, however, more information about the production parameters is gained. Hence, the uncertainty distributions need to be updated. As a consequence of this updating, the uncertain reservoir parameters V_i and D_i typically become stochastically dependent of each other even in cases where the two are independent apriori. This makes it very difficult to obtain analytical solutions. In order to find a solution to this issue, all the functions are estimated using Monte Carlo methods. In particular the updated joint distributions of V_i and D_i can be simulated using a combination of rejection sampling and the Metropolis-Hastings algorithm. The methodology for doing this is described in [8], so we skip the details here.

We now consider the kth period and assume that we have generated a random sample of size N from the updated joint distribution of V_i and D_i. We denote this sample by $(V_{i1}, D_{i1}), \dots, (V_{iN}, D_{iN})$. By the definitions of $\bar{F}_{ik}$ and $\bar{G}_{ik}$ it follows that these functions can be estimated by:

$$\bar{F}^{est}_{ik}(x_{ik}) = \frac{1}{N}\sum_{s=1}^{N} I(D_{is}(V_{is} - Q_{i,k-1}) > x_{ik}), \quad i = 1,\ldots,n, \quad k = 1,2,\ldots, \tag{21}$$

$$\bar{G}^{est}_{ik}(x_{ik}) = \frac{1}{N}\sum_{s=1}^{N} I(D_{is}(V_{is} - Q_{i,k-1}) > x_{ik}) \cdot D_{is}^{-1}, \quad i = 1,\ldots,n, \quad k = 1,2,\ldots, \tag{22}$$

respectively.

5. A NUMERICAL EXAMPLE

We illustrate the results of the previous sections by considering a simple numerical example with just two reservoirs, i.e., $n = 2$. Moreover, we assume that the reservoirs will be producing over 25 periods of time and processed on a facility with a capacity of $K = 1.2$ million barrels of oil per period.

The volumes, V_1 and V_2, associated with these reservoirs are both 12 million barrels of oil, while the decline rates D_1 and D_2, are 0.25 and 0.10 respectively. As these parameters are uncertain apriori, however, we assess prior probability distributions for all these quantities. The volumes are assumed to be lognormally distributed with expectation 12.0 and standard deviation 2.0, while the decline rates are assumed to be uniformly distributed over the intervals $[0.20, 0.30]$ and $[0.05, 0.15]$ respectively. This is summarized in Table 1. In addition to this, we assume that all reservoir parameters are stochastically independent of each other apriori.

Table 1: Reservoir parameters.

i	$E[V_i]$	$SD[V_i]$	V_i	D_i^{min}	D_i^{max}	D_i
1	12.0	2.0	12.0	0.20	0.30	0.25
2	12.0	2.0	12.0	0.05	0.15	0.10

In the simulation we compare three different strategies. Strategy 1 is the short-term strategy described in Section 3, while Strategy 2 is the long-term strategy described in Section 4. In addition to this we also consider a third strategy where we assume that the true values of all the reservoir parameters are known. As explained in Section 2 the optimal strategy in this situation is a strict priority rule where the reservoir with the lowest decline rate, i.e., Reservoir 2, is given top priority. This is of course not a realistic scenario, but this strategy is useful as a comparison to the other two.

Table 2: Results of the simulations for the three strategies.

	Total result	Disc. result
Strategy 1	22.15	20.26
Strategy 2	22.93	20.92
Strategy 3	22.94	20.94

In Table 2 we have listed the results of the simulations. The left-hand column contains the total volumes for the three strategies, while the right-hand column contains the corresponding discounted results obtained using a discount rate of 1 %. We observe that the long-term strategy (Strategy 2) is almost as good as the strict priority strategy (Strategy 3), while the short-term strategy (Strategy 1) produces noticable worse results.

We have also included three plots, Figure 1, Figure 2 and Figure 3, showing the production profiles of the three strategies. In all three plots the blue curve represents the total production, i.e., the sum of

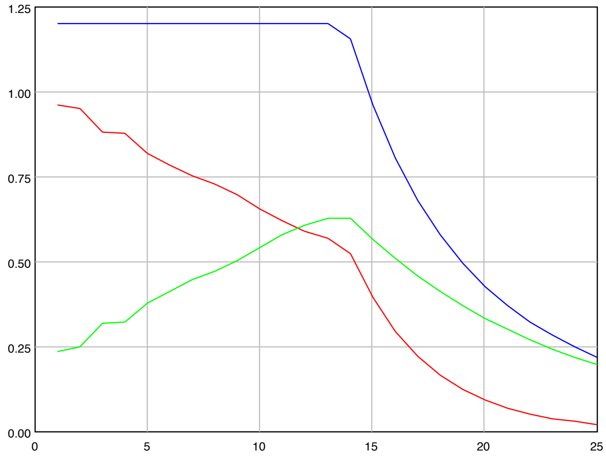

Figure 1: *Production profiles using short-term strategy under uncertainty for Reservoir 1 (red curve), Reservoir 2 (green curve), and Total production (blue curve)*

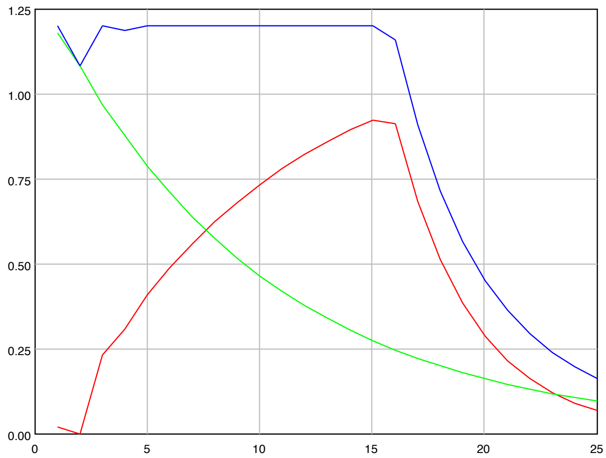

Figure 2: *Production profiles using long-term strategy under uncertainty for Reservoir 1 (red curve), Reservoir 2 (green curve), and Total production (blue curve)*

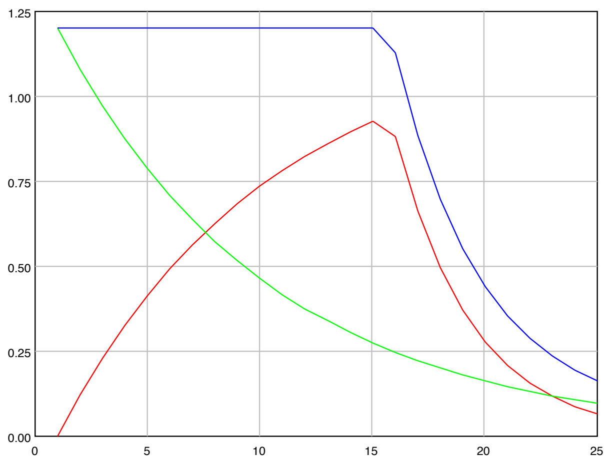

Figure 3: *Production profiles using optimal strategy with no uncertainty for Reservoir 1 (red curve), Reservoir 2 (green curve), and Total production (blue curve)*

the production from both reservoirs, while the red and green curves represent the production profiles of Reservoir 1 and 2 respectively.

Considering Figure 1, the plot representing the short-term strategy, we observe that Reservoir 1 is produced at a relatively *high* level in the beginning, while Reservoir 2 starts out a little lower. Since this strategy maximizes the probability of utilizing all the processing capacity, it makes sense to produce more from Reservoir 1, since this is the better reservoir.

Turning to Figure 2, we observe that using the long-term strategy, Reservoir 1 is produced at a very *low* level in the beginning. This is because that in order to obtain the best long-term results, the reservoirs with high decline rates should be saved as long as possible. In a short-term perspective, this strategy is more risky since there is a higher probability that Reservoir 2 may not be able to utilize as much of the processing capacity. As a result we see that there are a couple of dips in the toal production in the second and fourth periods. While these dips are unfortunate in a short-term perspective, such dips also have the side-effect that more information about the reservoir parameters are gained, and hence the uncertainty is reduced. In the long-term perspective this is an advantage. In fact, except for these dips, the long-term strategy produces production profiles that are very similar to the profiles of the optimal strategy shown in Figure 3. In particular, using the long-term strategy the plateau production is maintained in 15 periods, i.e., two periods more than if the short-term strategy is used. After that there is a steep decline in the production reflecting that the tail-production has a low center of mass.

6. CONCLUSIONS

In the present paper we have presented a framework for optimizing oil production from several reservoirs sharing a common processing facility when the reservoir parameters are not known. Using Lagrange optimization we have derived both a short-term strategy and a long-term strategy. However, both these strategies are determined using a step by step forward optimization procedure making the calculations simple and efficient compared to a full scale stochastic dynamic optimization. Numerical studies have shown that the long-term strategy is performing better than the short-term strategy. In fact in the numerical example shown in Section 5, the long-term strategy is almost as good as the optimal deterministic strategy. In an upcoming paper we will look further into this problem, and consider other more complex production models as well.

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