

Sequential optimization of oil production from multiple reservoirs under uncertainty

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Discrete time optimization under uncertainty

We consider the oil production from a field consisting of n reservoirs that share a processing facility with a constant process capacity K . The production from each reservoir is described as a discrete time process:

q_{ik} = The production from the i th reservoir in the k th period,

Q_{ik} = The cum. production from the i th reservoir after the k th period

$$= \sum_{j=1}^k q_{ij}$$

We also define $Q_{i0} = 0$.



Discrete time optimization under uncertainty

The maximum amount of oil that can be produced from the i th reservoir within the k th period given no other restrictions, is:

$$f_i(Q_{i,k-1}) = D_i(V_i - Q_{i,k-1}),$$

where $V_i > 0$ and $D_i \in [0, 1]$ are *random variables* and denote the *recoverable volume* and *decline rate* of the i th reservoir.

x_{ik} = quota assigned to the i th reservoir during the k th period

$$\mathbf{x}_k = (x_{1k}, \dots, x_{nk})$$

The actual production volumes are then given by:

$$q_{ik} = q_{ik}(x_{ik}) = \min\{f_i(Q_{i,k-1}), x_{ik}\},$$

where the quotas are chosen so that $\sum_{i=1}^n x_{ik} = K$.



Discrete time optimization under uncertainty

If $x_{ik} \leq f_i(Q_{i,k-1})$, it follows that $q_{ik} = x_{ik}$. If this holds for all reservoirs, all quotas are fully utilized, and we get that:

$$\sum_{i=1}^n q_{ik} = \sum_{i=1}^n x_{ik} = K.$$

If $x_{jk} > f_j(Q_{j,k-1})$ for some j , the quota for this reservoir cannot be fully utilized, i.e., $q_{jk} < x_{jk}$. Hence, in this case:

$$\sum_{i=1}^n q_{ik} < \sum_{i=1}^n x_{ik} = K.$$

A good production strategy should aim at utilizing the quotas as much as possible for all reservoirs.



Short-term optimization under uncertainty

In order to formulate the optimization problem, we introduce:

$$Y_k = Y_k(\mathbf{x}_k) = \sum_{j=1}^n q_{jk}(x_{jk}) = \sum_{j=1}^n \min\{f_j(Q_{j,k-1}), x_{jk}\}, \quad k = 1, 2, \dots$$

Considering the k th time period, the objective is to choose \mathbf{x}_k so that $E[Y_k(\mathbf{x}_k)]$ is maximized subject to the processing capacity constraint:

$$\sum_{i=1}^n x_{ik} = K.$$

NOTE: By using this approach at each step, the focus is on the upcoming time period only.

We refer to this optimization problem as the *short-term optimization problem*, and the solution will be called the *short-term production strategy*.



Short-term optimization under uncertainty

In order to solve the short-term optimization problem, we introduce the Lagrange function:

$$\Lambda_S(\mathbf{x}_k, \lambda) = \Phi_S(\mathbf{x}_k) - \lambda\Psi(\mathbf{x}_k),$$

where λ denotes the Lagrange multiplier, and where:

$$\begin{aligned}\Phi_S(\mathbf{x}_k) &= E[Y_k(\mathbf{x}_k)], \\ \Psi(\mathbf{x}_k) &= \sum_{i=1}^n x_{ik} - K.\end{aligned}$$

A stationary point for the Lagrange function is then found by solving the equation:

$$\nabla\Phi_S(\mathbf{x}_k) = \lambda\nabla\Psi(\mathbf{x}_k),$$

subject to the restriction that $\Psi(\mathbf{x}_k) = 0$.



Short-term optimization under uncertainty

For $i = 1, \dots, n$, we get:

$$\begin{aligned}\frac{\partial}{\partial x_{ik}} \Phi_S(\mathbf{x}_k) &= \frac{\partial}{\partial x_{ik}} E[Y_k(\mathbf{x}_k)] \\ &= E\left[\frac{\partial}{\partial x_{ik}} \sum_{j=1}^n \min\{f_j(Q_{j,k-1}), x_{jk}\}\right] \\ &= E\left[\frac{\partial}{\partial x_{ik}} \min\{f_i(Q_{i,k-1}), x_{ik}\}\right] \\ &= E[I(f_i(Q_{i,k-1}) > x_{ik})] \\ &= P(f_i(Q_{i,k-1}) > x_{ik}).\end{aligned}$$

where $I(\cdot)$ denotes the indicator function.

NOTE: $P(f_i(Q_{i,k-1}) > x_{ik})$ is a decreasing function of x_{ik} . This implies that Φ_S is a *concave* function. Thus, the stationary point will be a *maximum point*.



Short-term optimization under uncertainty

Finally, we get:

$$\nabla\Psi(\mathbf{x}_k) = (1, \dots, 1).$$

Combining all this, we get that the optimal solution must satisfy:

$$P(f_i(Q_{i,k-1}) > x_{ik}) = \lambda, \quad i = 1, \dots, n,$$

for some value of λ , as well as the processing capacity constraint:

$$\sum_{i=1}^n x_{ik} = K$$

NOTE: The short-term strategy attempts to distribute the available processing capacity between the reservoirs, and hence balance the risk between these.



Long-term optimization under uncertainty

In the deterministic case the optimal strategy is to make sure that the reservoirs with lowest decline rates are produced first. As a result, the tail-production will be dominated by the reservoirs with the highest decline rates. As a result the remaining volumes will be produced as fast as possible.

In order to improve the results for the stochastic case we introduce a different approach where more focus is put on the tail-production. In particular, we aim at finding a strategy where the tail-production can be done as fast as possible.



Long-term optimization under uncertainty

One way of evaluating the tail-production is by calculating its *potential center of mass*. We start out by considering the i th reservoir, and assume that we have completed $k - 1$ periods of production.

Assuming that the reservoir is produced at maximum speed in all the periods following the k th period, we get that:

$$q_{i,k+1} = D_i(V_i - Q_{i,k-1} - q_{ik}),$$

$$\begin{aligned} q_{i,k+2} &= D_i(V_i - Q_{i,k} - q_{i,k+1}) = D_i((V_i - Q_{i,k-1} - q_{ik}) - q_{i,k+1}) \\ &= D_i((V_i - Q_{i,k-1} - q_{ik}) - D_i(V_i - Q_{i,k-1} - q_{ik})) \\ &= D_i(1 - D_i)(V_i - Q_{i,k-1} - q_{ik}) \end{aligned}$$

$$\begin{aligned} q_{i,k+3} &= D_i(V_i - Q_{i,k+1} - q_{i,k+2}) = D_i((V_i - Q_{i,k} - q_{i,k+1}) - q_{i,k+2}) \\ &= D_i((V_i - Q_{i,k} - q_{i,k+1}) - D_i(V_i - Q_{i,k} - q_{i,k+1})) \\ &= D_i(1 - D_i)(V_i - Q_{i,k} - q_{i,k+1}) = D_i(1 - D_i)^2(V_i - Q_{i,k-1} - q_{ik}) \end{aligned}$$



Long-term optimization under uncertainty

For the $(k + h)$ -th period we get that:

$$q_{i,k+h} = D_i(V_i - Q_{i,k+h-2} - q_{i,k+h-1}) = D_i(1 - D_i)^{h-1}(V_i - Q_{i,k-1} - q_{ik})$$

The *potential center of mass* for the tail-production after the k th period of the i th reservoir, expressed as a function of x_{ik} , and denoted by $Z_{ik}(x_{ik})$, can now be defined as:

$$\begin{aligned} Z_{ik}(x_{ik}) &= \sum_{h=1}^{\infty} h \cdot q_{i,k+h} \\ &= \sum_{h=1}^{\infty} h \cdot D_i(1 - D_i)^{h-1}(V_i - Q_{i,k-1} - q_{ik}(x_{ik})) \\ &= D_i(V_i - Q_{i,k-1} - q_{ik}(x_{ik})) \sum_{h=1}^{\infty} h \cdot (1 - D_i)^{h-1} \end{aligned}$$



Long-term optimization under uncertainty

In order to determine the infinite sum we introduce:

$$g(D_i) = \sum_{h=1}^{\infty} (1 - D_i)^h = \frac{1 - D_i}{D_i},$$

where the last equation follows by the formula for the sum of a geometric series. Hence, by differentiating this equation with respect to D_i , it follows that:

$$g'(D_i) = - \sum_{h=1}^{\infty} h(1 - D_i)^{h-1} = -D_i^{-2}$$

Thus, it follows that:

$$\begin{aligned} Z_{ik}(x_{ik}) &= D_i(V_i - Q_{i,k-1} - q_{ik}(x_{ik})) \sum_{h=1}^{\infty} h \cdot (1 - D_i)^{h-1} \\ &= (V_i - Q_{i,k-1} - q_{ik}(x_{ik})) \cdot D_i^{-1} \end{aligned}$$



Long-term optimization under uncertainty

The potential center of mass for the tail-production after the k th period of all reservoirs combined, is defined as:

$$Z_k(\mathbf{x}_k) = \sum_{j=1}^n Z_{jk}(x_{jk}) = \sum_{j=1}^n (V_j - Q_{j,k-1} - q_{jk}(x_{jk})) \cdot D_j^{-1}.$$

We seek a strategy where the expected potential center of mass is as low as possible, since this implies that the remaining volumes can be produced as fast as possible.

We refer to this optimization problem as the *long-term optimization problem*, and the solution will be called the *long-term production strategy*.



Long-term optimization under uncertainty

In order to solve the long-term optimization problem, we again introduce the Lagrange function:

$$\Lambda_L(\mathbf{x}_k, \lambda) = \Phi_L(\mathbf{x}_k) - \lambda\Psi(\mathbf{x}_k),$$

where λ denotes the Lagrange multiplier, and where:

$$\Phi_L(\mathbf{x}_k) = E[Z_k(\mathbf{x}_k)],$$

$$\Psi(\mathbf{x}_k) = \sum_{i=1}^n x_{ik} - K.$$

A stationary point for the Lagrange function is then found by solving the equation:

$$\nabla\Phi_L(\mathbf{x}_k) = \lambda\nabla\Psi(\mathbf{x}_k),$$

subject to the restriction that $\Psi(\mathbf{x}_k) = 0$.



Long-term optimization under uncertainty

For $i = 1, \dots, n$ we get:

$$\begin{aligned}\frac{\partial}{\partial x_{ik}} \Phi_L(\mathbf{x}_k) &= \frac{\partial}{\partial x_{ik}} E[Z_k(\mathbf{x}_k)] \\ &= E\left[\frac{\partial}{\partial x_{ik}} Z_k(\mathbf{x}_k)\right] \\ &= E\left[\frac{\partial}{\partial x_{ik}} \sum_{j=1}^n (V_j - Q_{j,k-1} - q_{jk}(x_{jk})) \cdot D_j^{-1}\right] \\ &= -E\left[\frac{\partial}{\partial x_{ik}} \min\{f_i(Q_{i,k-1}), x_{ik}\} \cdot D_i^{-1}\right] \\ &= -E[I(f_i(Q_{i,k-1}) > x_{ik}) \cdot D_i^{-1}].\end{aligned}$$

NOTE: $-E[I(f_i(Q_{i,k-1}) > x_{ik}) \cdot D_i^{-1}]$ is an increasing function of x_{ik} . This implies that Φ_L is a *convex* function. Thus, the stationary point will be a *minimum point*.

Long-term optimization under uncertainty

Finally, as in the previous case, we get:

$$\nabla \Psi(\mathbf{x}_k) = (1, \dots, 1).$$

Combining all this, we get that the optimal solution must satisfy:

$$E[I(f_i(Q_{i,k-1}) > x_{ik}) \cdot D_i^{-1}] = -\lambda, \quad i = 1, \dots, n,$$

for some value of λ , as well as the processing capacity constraint:

$$\sum_{i=1}^n x_{ik} = K.$$

NOTE: The long-term strategy tends to give priority to reservoirs with smaller decline rates.



Handling the uncertainty

- By assessing distributions for the reservoir parameters, the expected values needed in the calculations of the optimal solutions can be computed.
- As the production develops, more information about the production parameters is gained. Hence, the uncertainty distributions must be updated (using Bayes' theorem).
- As a result of the updating, V_i and D_i typically become stochastically dependent even when they are independent a priori.
- The updated joint distributions of V_i and D_i can be simulated using a combination of rejection sampling and the *Metropolis-Hastings algorithm*.



A numerical example

We consider a simple numerical example, where $n = 2$, and where V_1 and V_2 are lognormally distributed apriori, while D_1 and D_2 are uniformly distributed apriori.

The reservoirs will be producing in 25 periods and processed on a facility with a capacity of $K = 1.2$ million barrels of oil per period.

i	$E[V_i]$	$SD[V_i]$	V_i	D_i^{min}	D_i^{max}	D_i
1	12.0	2.0	12.0	0.20	0.30	0.25
2	12.0	2.0	12.0	0.05	0.15	0.10

Table: Reservoir parameters.

Given the true values of the reservoir parameters, the optimal strategy is a *strict priority rule* where the reservoir with the lowest decline rate, i.e., Reservoir 2, is given top priority.



Results

	Total result	Disc. result
Short-term strategy	22.15	20.26
Long-term strategy	22.93	20.92
Deterministic strategy	22.94	20.94

Table: Results of the simulations for the three strategies.



Production profiles for the short-term strategy

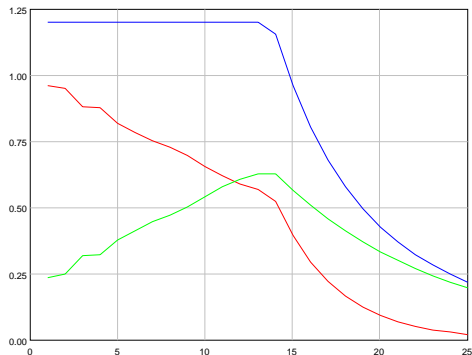


Figure: Production profiles using short-term strategy for Reservoir 1 (red curve), Reservoir 2 (green curve), and Total production (blue curve)



Production profiles for the long-term strategy

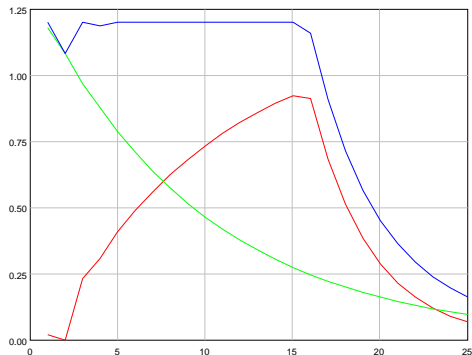


Figure: Production profiles using long-term strategy for Reservoir 1 (red curve), Reservoir 2 (green curve), and Total production (blue curve)



Production profiles for the deterministic strategy

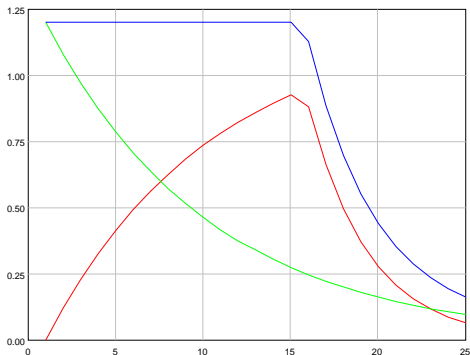


Figure: Production profiles using the deterministic strategy for Reservoir 1 (red curve), Reservoir 2 (green curve), and Total production (blue curve)



Conclusions

- A framework for optimizing oil production from several reservoirs sharing a common processing facility when the reservoir parameters are not known is proposed
- Both a short-term strategy and a long-term strategy have been analysed
- Both strategies are determined using step by step forward optimization making the calculations simple and efficient compared to full scale stochastic dynamic optimization
- Numerical studies have shown that the long-term strategy is performing better than the short-term strategy.
- Future work:
 - Multiphase production (i.e., oil/gas/water)
 - More complex production models

