UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in: STK4500 — Life insurance and finance

Day of examination: Friday June 16 2017

Examination hours: 14.30 – 18.30

This problem set consists of 3 pages.

Appendices: None

Permitted aids: Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Comments: Most sub-problems can be solved independently of each other. Problem 2 c) might be the most difficult one and is perhaps the one you should skip if time is short. Try to submit answers in intelligble handwriting!

Problem 1 Pension insurance

Let q_l be the probability that an individual of age l dies the coming year.

a) Explain how a life table ${}_kp_l$ is constructed for $l = l_0, \ldots, l_e$ where l_0 is a minimum age and l_e a maximum.

Consider an individual who at age l_r decides to retire with his/her savings v_0 used to purchase a fixed pension s received at the start of each year lasting to age l_e.

b) Determine s when calculated from the equivalence principle. The discount is $d = 1/(1+r)$ where r is the technical rate of interest.

c) Offer a mathematical expression for how much the individual receives if the pension is sold back to the insurance company after k years when he or she is still alive.

Suppose the pension instead of v_0 was financed by a fixed, annual premium π up to the retirement age with payments in advance from the age l_0.

d) Write down a mathematical expression for the equivalence π. How does it relate to v_0?

Suppose the individual wants more money in the beginning of the pension period than later when he/she has become old. One way to express such a wish mathematically is to

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assume that the pension at time $l_r + k$ is of the form

$$s_k = s_0 \times e^{-\omega k}, \qquad k = 0, 1, \ldots$$

where ω is a known decay rate (for example $\omega = 0.05$ or $\omega = 0.10$).

e) Determine s_0 when the pension is still adapted to the savings v_0 at age l_r by the principle of equivalence and explain how the expression for the repurchase value in c) must be modified.

Problem 2 Solvency II

a) How is the Basic Own Funds (BOF) in Solvency II defined from the value of the assets A, the Best Estimate BE and the risk margin RM?

b) What is the difference beteeen the discount schemes used in traditional life insurance as in Problem 1 and the discount schemes used in Solvency II?

Consider a pension portfolio similar to the one in Problem 1b) with fixed pensions $s_1, \ldots, s_n$ determined from original savings $v_{01}, \ldots, v_{0n}$.

c) Write down an expression for the Best Estimate BE at a point in time when these pensions have lasted $k_1, \ldots, k_n$ years with all policy holders still alive [**Hint:** Utilize Problem 1c)].

d) What is a stress test? Propose a stress model for the mortalities q_l when used to examine the uncertainty of the pension portfolio.

e) Explain how the stressed value BE^s of the Best Estimate is calculated under the stress model in d) and how it is used to determine the Solvency Capital Requirement (SCR) for longevity for the pension portfolio above.

Problem 3 Term insurance under inflation

Consider a term insurance contract lasting K years with a one-time payment s in arrears at the end of the year the policy holder has died and with no repayment back from the company if he/she lives through the entire period K. The scheme is financed by a single payment (premium) V_0 at the beginning when the age of the policy holder is l_0. The technical rate of interest is r.

a) Which are the probabilities ${}_kq_l$ needed to value such schemes? Explain how they are calculated from the life table ${}_kp_l$ in Problem 1.

b) Write down a mathematical expression for V_0 when it is the present value of all future payments.

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During the rest of this problem it is assumed that the contract is indexed-linked so that a payment at the end of period k is $Q_k \times s$ with Q_k the price level at that time with $Q_0 = 1$ at the beginning. Assume the Wilkie inflation model which specifies the price changes as

$$Q_k = (1 + I_k)Q_{k-1} \qquad \text{where} \qquad I_k = (1 + \xi)e^{X_k} - 1, \qquad X_k = aX_{k-1} + \sigma\varepsilon_k$$

for $k = 1, \ldots, K$. Here $\varepsilon_1, \ldots, \varepsilon_K$ is an independent sequence of normal variables with mean 0 and standard deviation 1. The initial inflation $I_0 = \xi$. There are three parameters ξ, σ and a.

c) Modify the mathematical expression in b) so that it captures a fixed rate of inflation $I_k = \xi$. How can you adapt the Wilkie model to that situation?

d) Sketch a computer progam that simulates the price level sequence $Q_1, \ldots, Q_K$ and explain how the present value V_0 of the payments at the beginning can be simulated.

Consider the four different inflation scenarios

Scenario 1	Scenario 2	Scenario 3	Scenario 4
$(\xi, \sigma) = (0, 0)$	$(\xi, \sigma) = (0.02, 0.01)$	$(\xi, \sigma) = (0.03, 0.01)$	$(\xi, \sigma) = (0.02, 0.02)$.

The other parameters are $a = 0.5$, $s = 1$, $r = 0.02$ with the mortalities those for US males in the oblig. The table below summarizes mean, standard deviation and 0.5% and 99.5% percentiles for V_0 under the four scenarios, but **recorded in a different order**:

	mean	sd	0.5%	99.5%
Calculation 1	0.116	0.058	0.039	0.373
Calculation 2	0.125	0.029	0.070	0.226
Calculation 3	0.106	0.024	0.060	0.188
Calculation 4	0.056	0	0.056	0.056

The number of simulations used was one million.

e) Interprete the Scenario 1 model $(\xi, \sigma) = (0, 0)$. Which of the four calculations does it correspond to?

f) Which model scenario corresponds to which calculation? Justify your answer and propose for each scenario an equivalence price V_0 for the insurance arrangement.

End