

Fasit 2004

Parametre

```
In[1]:= << "Statistics`ContinuousDistributions`"  
       << "Graphics`Graphics`"  
       << "Graphics`Arrow`"
```

```
In[4]:= gKPI = 1.02;  
       gG = 1.04;  
        $\mu$  = Log[gG + 0.03];  
        $\sigma$  = 0.05;  
        $\omega$  = 120;  
       i = 0.06;
```

Simuleringsfunksjon

```
In[10]:= laggLOF =  
       Compile[{{atc, _Real, 1}, {antallUtbetalinger, _Integer}, {gGc, _Real}},  
       Module[{gLOF = Table[0., {antallUtbetalinger}]},  
       gLOF[[1]] = Min[gGc, 1 + Max[atc[[1]] - 1.03, 0]];  
       Do[gLOF[[t]] = Min[gGct, gLOF[[t - 1]] (1 + Max[atc[[t]] - 1.03, 0])], {t, 2, 3}];  
       Do[gLOF[[t]] = Min[gLOF[[t - 3]] gGc3, gLOF[[t - 1]] (1 + Max[atc[[t]] - 1.03, 0])],  
       {t, 4, antallUtbetalinger}]; gLOF];
```

b)

```
In[11]:= n1 = 10; (* Antall utbetalinger *)  
       n2 = 100000; (* Antall simuleringer *)
```

Legger de simulerte avkastningene i en $n_2 \times n_1$ matrise:

```
In[13]:= atSim = Partition[e $\mu + \sigma$  RandomArray[NormalDistribution[0,1],n1 n2], n1];
```

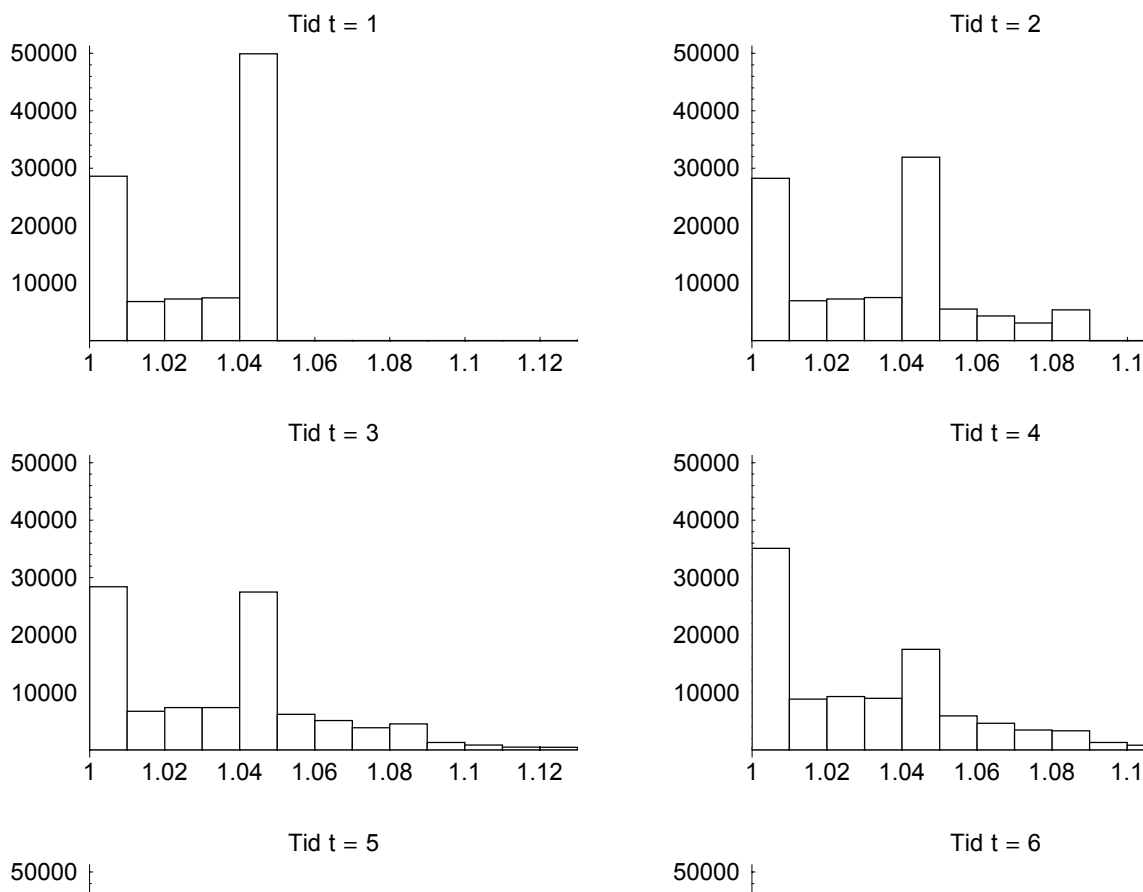
```
In[14]:= gLOF = Table[laggLOF[atSim[[j]], n1, gG], {j, n2}];
```

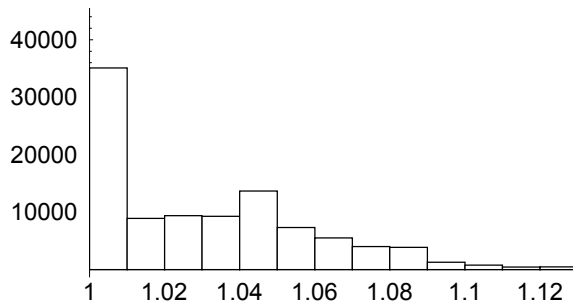
```
In[15]:= årligReguleringLOF = Transpose[Table[
  Prepend[Table[ $\frac{gLOF[[j, t]]}{gLOF[[j, t - 1]]}$ , {t, 2, n1}], gLOF[[j, 1]], {j, 1, n2}]]];
```

Sannsynlighetsfordeling:

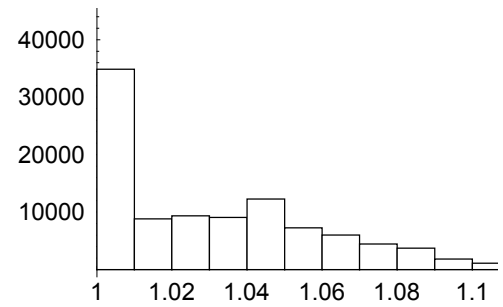
```
In[16]:= visHistogramÅrligReguleringLOF[tid_] :=
  Show[Histogram[årligReguleringLOF[[tid]], HistogramCategories →
    Table[k, {k, 1 -  $\frac{1}{10^9}$ , 1.13, .01}], BarStyle → RGBColor[1, 1, 1],
    HistogramRange → {1, gG3}, DisplayFunction → Identity],
  Graphics[{PointSize[ $\frac{1}{10^9}$ ], Point[{1,  $\frac{n_2}{2}$ }]},
  DisplayFunction → Identity, DefaultFont → {"Helvetica", 9},
  PlotLabel → "Tid t = " <> ToString[tid]]];
```

```
In[20]:= Display["P:STK4500/Oppgaver/Oppgavesamling/b_Histogram.eps",
  Show[GraphicsArray[Partition[Table[visHistogramÅrligReguleringLOF[tid],
    {tid, 1, 10}], 2]], ImageSize → 500], "EPS"];
```

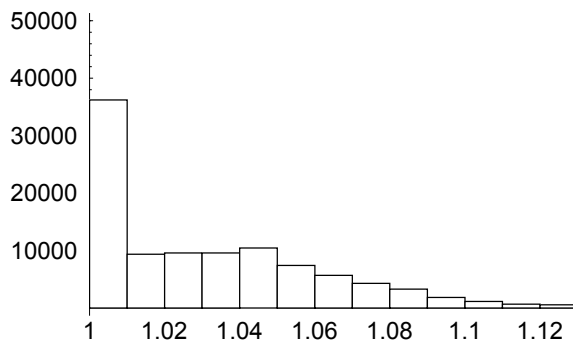




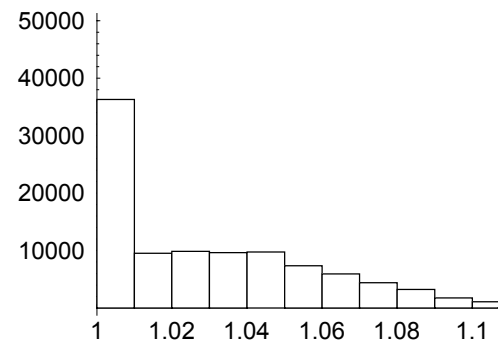
Tid t = 7



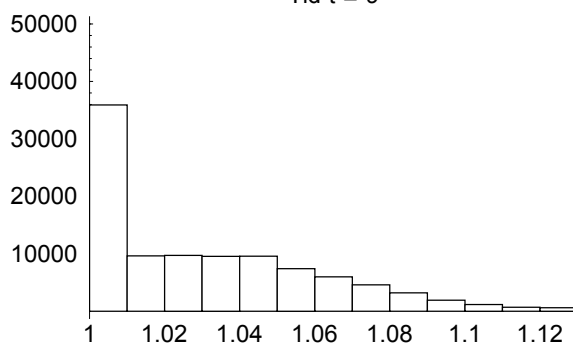
Tid t = 8



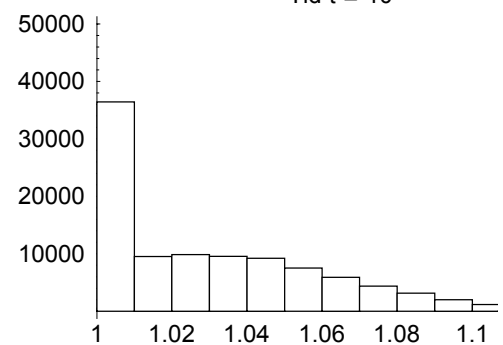
Tid t = 9



Tid t = 10



Tid t = 9

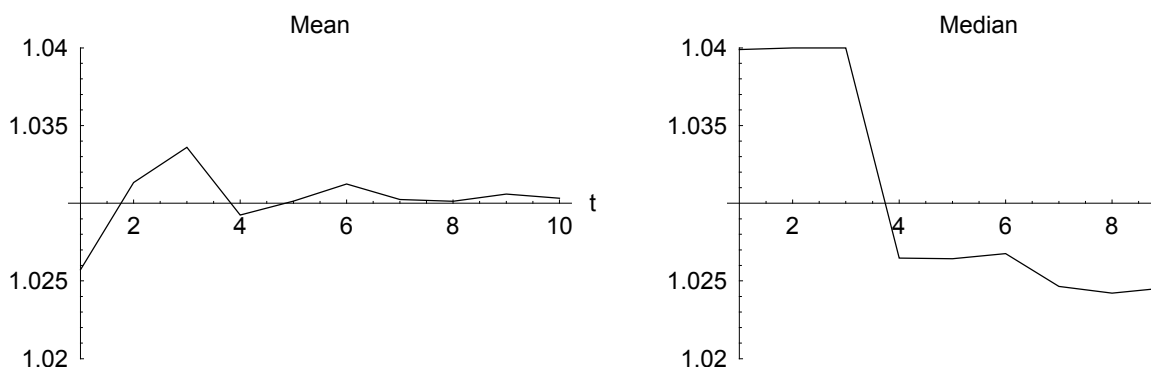


Tid t = 10

Mean og median:

```
In[18]:= visPlottÅrligReguleringLOF[egenskap_] :=
  ListPlot[Table[egenskap[ÅrligReguleringLOF[[j]]], {j, n1}],
  PlotJoined → True, DisplayFunction → Identity, PlotRange → {1.02, 1.04},
  PlotLabel → ToString[egenskap], DefaultFont → {"Helvetica", 9},
  AxesLabel → {"t", None}, AxesOrigin → {1, 1.03}];
```

```
In[22]:= Display["P:STK4500/Oppgaver/Oppgavesamling/b_Mean_Median.eps",
  Show[GraphicsArray[visPlottÅrligReguleringLOF[#1] & /@ {Mean, Median}],
  ImageSize -> 500], "EPS"];
```

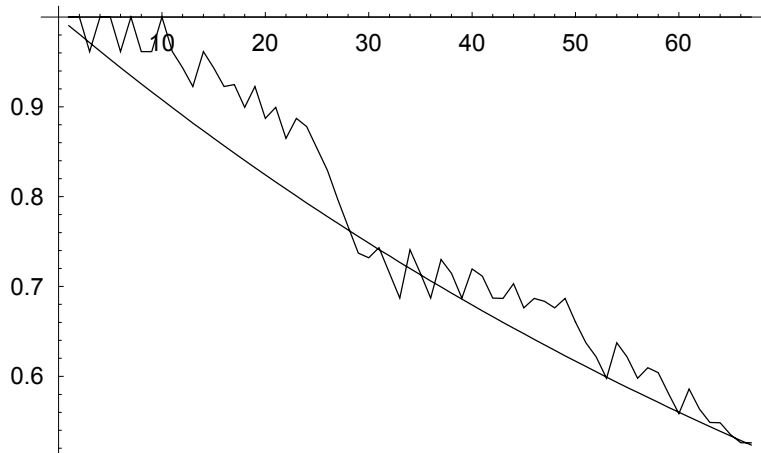


c)

n er utbetalingstidens lengde

```
In[23]:= simulerBaner[n_] := Module[{},
  gLOF = laggLOF[eμ+σ RandomArray[NormalDistribution[0,1],n], n, gG];
  gFT = Table[gGt, {t, n}];
  gModFT = Table[( $\frac{gG + gKPI}{2}$ )t, {t, n}];
  Show[(ListPlot[#1, PlotJoined -> True,
    DefaultFont -> {"Helvetica", 9}, DisplayFunction -> Identity] &) /@
    { $\frac{gFT}{gFT}$ ,  $\frac{gLOF}{gFT}$ ,  $\frac{gModFT}{gFT}$ }, DisplayFunction -> $DisplayFunction];
```

```
In[24]:= Display["P:STK4500/Oppgaver/Oppgavesamling/c_baner.eps",
  simulerBaner[67], "EPS"];
```



Underreguleringen gjør seg mer gjeldende i dette lange tidsperspektivet. Uføre pensjonisten får lite å rutt med som alderspensjonist!

d)

```
In[25]:= n1 = 30; (* Antall utbetalinger *)
n2 = 1000; (* Antall simuleringer *)
```

Legger de simulerte avkastningene i en n2 x n1 matrise:

```
In[27]:= atSim = Partition[eμ+σ RandomArray[NormalDistribution[0,1],n1 n2], n1];
```

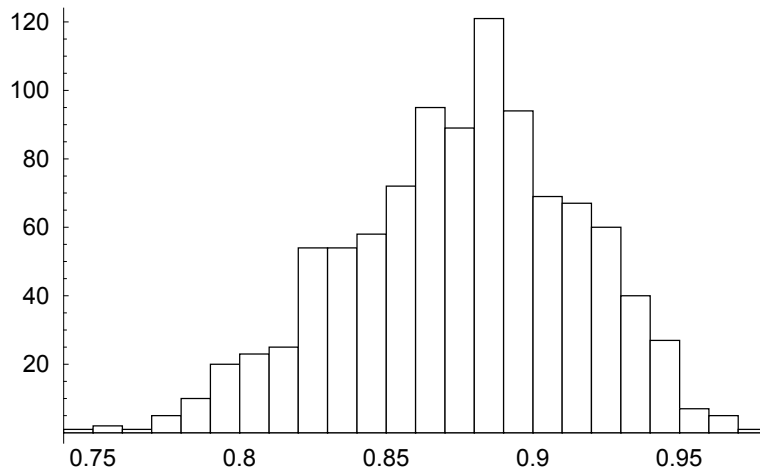
```
In[28]:= gLOF = Table[laggLOF[atSim[[j]], n1, gG], {j, n2}];
```

```
In[29]:= forventetKontantverdiG =  $\sum_{j=1}^{30} \left(\frac{1}{1+i}\right)^j gG^j$ ;
```

```
forventetKontantverdiGKPI =  $\sum_{j=1}^{30} \left(\frac{1}{1+i}\right)^j \left(\frac{gG + gKPI}{2}\right)^j$ ;
```

```
In[31]:= kontantverdierLOF = Table[ $\sum_{j=1}^{n1} \frac{\left(\frac{1}{1+i}\right)^j gLOF[[k, j]]}{forventetKontantverdiG}$ , {k, 1, n2}];
```

```
In[32]:= Display["P:STK4500/Oppgaver/Oppgavesamling/d_Histogram.eps",
Histogram[kontantverdierLOF, DefaultFont -> {"Helvetica", 9},
HistogramCategories -> Table[k, {k, 0, 1, .01}],
BarStyle -> RGBColor[1, 1, 1]], "EPS"];
```



```
In[33]:= Print["Neddiskontert verdi av LOF-ytelsene
målt som enheter av den neddiskonterte verdien av
den G-reg. ytelsen: ", Mean[kontantverdierLOF]];
Print["Neddiskontert verdi av G/KPI-ytelsene målt som enheter
av den neddiskonterte verdien av den G-reg. ytelsen: ",
forventetKontantverdiGKPI
forventetKontantverdiG ];
```

Neddiskontert verdi av LOF-ytelsene målt som enheter av
den neddiskonterte verdien av den G-reg. ytelsen: 0.875687

Neddiskontert verdi av G/KPI-ytelsene målt som enheter av
den neddiskonterte verdien av den G-reg. ytelsen: 0.875792

e)

Sjekker formelen for overlevelsessannsynligheten:

$$\text{In[35]}:= \int a^{\tau} d\tau$$

$$\text{Out[35]}:= \frac{a^{\tau}}{\text{Log}[a]}$$

```
In[36]:= Clear[\alpha, \beta, c];
```

$$\text{Simplify}\left[\text{Exp}\left[-\int_0^{\tau} \left(\alpha + \beta c^{x+\tau-\frac{a+\tau}{b}}\right) d\tau\right]\right]$$

$$\text{Out[37]}:= e^{-\tau} \alpha + \frac{b c^{-\frac{a+\tau-b}{b}} (-c^{\tau} + c^{\frac{\tau}{b}}) \beta}{(-1+b) \text{Log}[c]}$$

f)

Sjekker utledet formel for median når $\alpha = 0$:

```
In[38]:= Clear[α, β, c];
Simplify[Solve[e- $\frac{\beta(c^t-1)}{\text{Log}[c]}$ ] ==  $\frac{1}{2}$ , t, InverseFunctions -> True][[1, 1, 2]]
Out[39]:=  $\frac{\text{Log}\left[1 + \frac{\text{Log}[2] \text{Log}[c]}{\beta}\right]}{\text{Log}[c]}$ 
```

Forventet levetid og median:

```
In[40]:= {α, β, c} = {0, .0000014, 1.14};
In[41]:= p[a_, b_, x_, t_] := e-α t -  $\frac{(\beta c^{x-\frac{a}{\beta}})(c^{(1-\frac{1}{\beta})^t}-1)}{(1-\frac{1}{\beta}) \text{Log}[c]}$ ];
In[42]:= Print["Forventning: ",
NIntegrate[Limit[p[0, b, 0, τ], b -> ∞], {τ, 0, ω}], " år"];
Print["Median: ",  $\frac{\text{Log}\left[1 + \frac{\text{Log}[2] \text{Log}[c]}{\beta}\right]}{\text{Log}[c]}$ , " år"]
Forventning: 82.9562 år
Median: 84.5634 år
```

g)

```
In[44]:= ekDT[{a_, b_}, gR_, x_] := NIntegrate[ $\left(\frac{gR}{1+i}\right)^\tau$  p[a, b, x, τ], {τ, 0, ω - x}];
```

```
In[45]:= Display["P:STK4500/Oppgaver/Oppgavesamling/g_Plott.eps",  
Plot[ $\frac{\text{ekDT}[\{0, 10^6\}, \text{gG}, 67]}{\text{ekDT}[\{a, 10\}, \frac{\text{gG}+\text{gKPI}}{2}, 67]}$ , {a, 0, 50},  
DefaultFont -> {"Helvetica", 9}, AxesLabel -> {"a", None}], "EPS"];
```

