Oppgave 7

Løsning

```
In[1]:=   {α, β, c} = {0, 0.0000202, 1.1015};
          i = 0.03;
          ξ = 0.7;
          v = 1/(1 + i);
          x = 30;
          n = 35;
          ω = 120;
          λ = 0.03;
          θ = 0.015;
          lønn = 500000;
          μ = 0.057;
          σ = 0.056;
```

```
In[13]:=  << "Statistics`ContinuousDistributions`"
          << "Graphics`Legend`"
          << "Graphics`Graphics`"
```

$$In[16]:=\quad p[y\_, t\_] := e^{-\left(\alpha\, t + \frac{\beta\, c^y\, (c^t - 1)}{Log[c]}\right)};$$

$$In[17]:=\quad a[t\_] := \sum_{j=n-t}^{\omega-(x+t)} v^j\, p[x + t, j];$$

Tabellerer funksjonene for de verdiene som brukes. Tidsbesparende: Hente verdi i tabell tar kortere tid enn et funksjonskall.

```
In[18]:=  at = Table[a[t], {t, 0, n - 1}];
          px = Table[p[x, t], {t, 0, n - 1}];
```

> **nSim** = antall simuleringer
>
> **ran1** = (n-1) x nSim matrise med N(0,1)-simuleringer
>
> **ran2** = n x nSim matrise med N(0,1)-simuleringer
>
> **simVt** = simulert premiereserve
>
> **simFt** = simulert forsikringsfond uten garanti
>
> **simFtG** = simulert forsikringsfond med garanti
>
> **innbet** = simulerte innbetalinger uten garanti
>
> **innbetG** = simulerte innbetalinger med garanti

```
In[20]:=  nSim = 1000;
```

```
In[21]:=  ran1 = Partition[RandomArray[NormalDistribution[0, 1], nSim (n - 1)], n - 1];
          ran2 = Partition[RandomArray[NormalDistribution[0, 1], nSim n], n];
```

```
In[23]:=  simInnbet[sim_] := Module[{simL, simA, simP, simS, simVt, simFt},
```

$$simL = FoldList[(1 + \lambda) \#1 + \theta \#2 \#1 \&, \text{lønn}, ran1[\![sim]\!]];$$

$$simS = 0.2\, simL;$$

$$simP = Prepend\Big[Table\Big[\Big(\frac{(t + 1)\, simS[\![t + 1]\!]}{n} - \frac{t\, simS[\![t]\!]}{n}\Big)\, at[\![t + 1]\!], \{t, 1, n - 1\}\Big],$$

$$\frac{simS[\![1]\!]\, at[\![1]\!]}{n}\Big]; \quad simA = e^{\mu - \frac{\sigma^2}{2} + \sigma\, ran2[\![sim]\!]};$$

$$simVt = Table\Big[\Big\{t, \frac{(t + 1)\, simS[\![t + 1]\!]\, at[\![t + 1]\!]\, px[\![t + 1]\!]}{n}\Big\}, \{t, 0, n - 1\}\Big];$$

$$simFt = Table[\{t + 1, simVt[\![t + 1, 2]\!]\, simA[\![t + 1]\!]\}, \{t, 0, n - 1\}]; \quad simFtG =$$

$$\quad Table[\{t + 1, simVt[\![t + 1, 2]\!]\, ((1 + i) + \xi\, Max[0, simA[\![t + 1]\!] - (1 + i)])\}, \{t, 0, n - 1\}];$$

$$innbet = Prepend[Table[simVt[\![t + 1, 2]\!] - simFt[\![t, 2]\!], \{t, 1, n - 1\}], simP[\![1]\!]];$$

$$innbetG = Prepend[Table[simVt[\![t + 1, 2]\!] - simFtG[\![t, 2]\!], \{t, 1, n - 1\}], simP[\![1]\!]];$$

$$\{innbet, innbetG\}\Big];$$

> Mulige baner for faktisk nødvendig innbetaling med og uten garanti:

```
In[24]:=  innbet = simInnbet[3];
```

```
In[25]:=  plot1 = Plot[{innbet[[1, Round[k]]], innbet[[2, Round[k]]]}, {k, 1, 35}, PlotRange → All,
              DefaultFont → {"Helvetica", 11}, PlotStyle → (RGBColor[#1, 0, 0] &) /@ {0, 1},
              PlotLegend → {"Uten garanti", "Med garanti"},
              LegendPosition → {-.5, .1}, LegendTextSpace → 5];
```
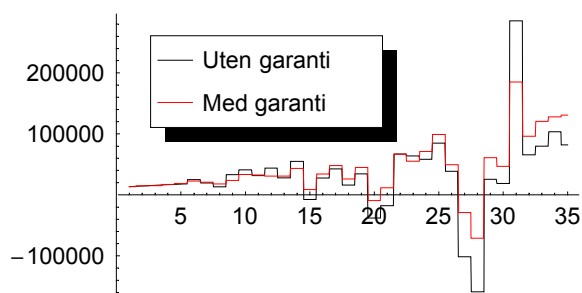
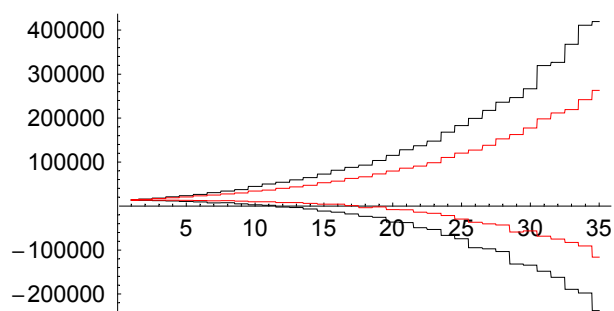*In[26]:=* `(*Display["p:STK4500/Oppgaver/Oppgave7/plot1.eps",Show[plot1],"EPS"];*)`

Trakt:

In[27]:= `Timing[innbetTab = Table[simInnbet[j], {j, nSim}];]`

Out[27]= `{3.615 Second, Null}`

In[28]:= `trakt = ({Take[Sort[#1], {0.05 nSim}], Take[Sort[#1], {0.95 nSim}]} &) /@`
        `Transpose[Transpose[innbetTab]〚1〛];`
    `traktG = ({Take[Sort[#1], {0.05 nSim}], Take[Sort[#1], {0.95 nSim}]} &) /@`
        `Transpose[Transpose[innbetTab]〚2〛];`

In[30]:= `plot2 = Plot[{trakt〚Round[k], 1〛, trakt〚Round[k], 2〛,`
        `traktG〚Round[k], 1〛, traktG〚Round[k], 2〛}, {k, 1, 35}, PlotRange → All,`
   `DefaultFont → {"Helvetica", 11}, PlotStyle → (RGBColor[#1, 0, 0] &) /@ {0, 0, 1, 1}];`

In[31]:= `(*Display["p:STK4500/Oppgaver/Oppgave7/plot2.eps",Show[plot2],"EPS"];*)`

Ekstra: Animering (Ctrl y). Ser ikke bare på 5 % og 95 % kvantiler som over, men på 5 %, 10 %, ..., 95 %.

In[32]:= `trakt = Table[(Take[Sort[#1], {` $\frac{pr\, nSim}{100}$ `}] &) /@`
        `Transpose[Transpose[innbetTab]〚1〛], {pr, 5, 95, 5}];`
    `traktG = Table[(Take[Sort[#1], {` $\frac{pr\, nSim}{100}$ `}] &) /@ Transpose[Transpose[innbetTab]〚2〛],`
        `{pr, 5, 95, 5}];`

```
In[34]:= Do[Plot[{trakt〚j, Round[k]〛, traktG〚j, Round[k]〛}, {k, 1, 35},
            PlotRange → {-200000, 400000}, DefaultFont → {"Helvetica", 11},
            PlotStyle → (RGBColor[#1, 0, 0] &) /@ {0, 1},
            PlotLabel → ToString[5 * j] <> " prosent kvantil"], {j, 1, 19}];
```
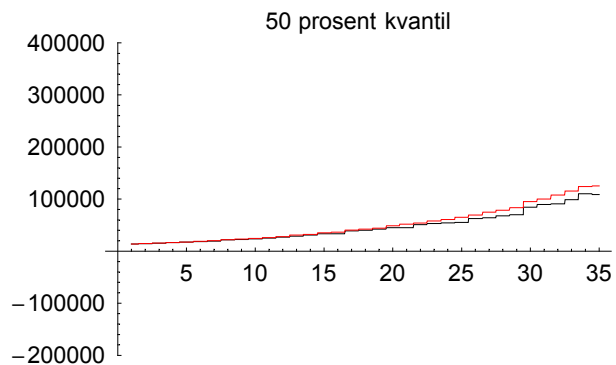

Kontantverdi til innbetalingene med og uten garanti:

```
In[35]:= simAvkastning = Table[e^(μ - σ²/2 + σ ran2〚j〛), {j, 1, nSim}];
```

$$\text{In[36]:= } \mathtt{kontantVerdi = Table}\Big[$$

$$\mathtt{innbetTab〚j, 1〛 . Delete}\Big[\mathtt{FoldList}\Big[\mathtt{\#2\ \#1\ \&,\ 1,\ }\frac{1}{\mathtt{simAvkastning〚j〛}}\Big]\mathtt{,\ n}\Big]\mathtt{,\ \{j,\ 1,\ nSim\}}\Big];$$

$$\mathtt{kontantVerdiG = Table}\Big[\mathtt{innbetTab〚j, 2〛 .}$$

$$\mathtt{Delete}\Big[\mathtt{FoldList}\Big[\mathtt{\#2\ \#1\ \&,\ 1,\ }\frac{1}{\mathtt{simAvkastning〚j〛}}\Big]\mathtt{,\ n}\Big]\mathtt{,\ \{j,\ 1,\ nSim\}}\Big];$$

```
In[38]:= plot3 =
```

$$\mathtt{Show}\Big[\mathtt{Histogram}\Big[\frac{\mathtt{kontantVerdi}}{10^6}\mathtt{,\ HistogramCategories → Table[i / 30, \{i, 0, 1000\}],}$$

$$\mathtt{HistogramRange → \{0, 2\}, DisplayFunction → Identity}\Big]\mathtt{,}$$

$$\mathtt{Histogram}\Big[\frac{\mathtt{kontantVerdiG}}{10^6}\mathtt{,\ HistogramCategories → Table[i / 30, \{i, 0, 1000\}],}$$

$$\mathtt{HistogramRange → \{0, 2\}, DisplayFunction → Identity,}$$

$$\mathtt{BarStyle → RGBColor[1, 1, 1]}\Big]\mathtt{,\ DisplayFunction → \$DisplayFunction,}$$

$$\mathtt{DefaultFont → \{"Helvetica", 11\}, Ticks → \{Automatic, False\}}\Big];$$
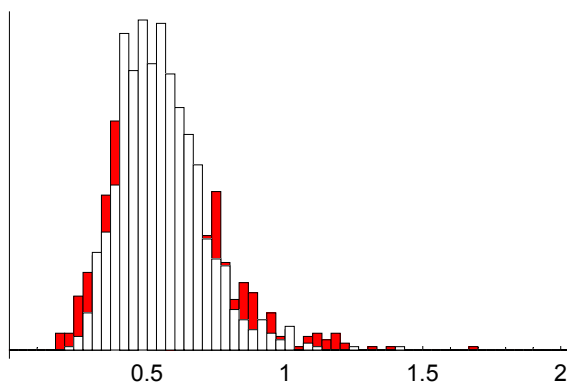

```
In[39]:= (*Display["p:STK4500/Oppgaver/Oppgave7/plot3.eps",Show[plot3],"EPS"];*)
```

In[40]:= `Show[Histogram[` $\frac{\text{kontantVerdi}}{\text{kontantVerdiG}}$ `[[Range[#1]]], DefaultFont → {"Helvetica", 11},`

`HistogramCategories → Table[` $\frac{\text{i}}{20}$ `, {i, 0, 100}], HistogramRange → {.5, 1.5},`

`DisplayFunction → Identity], Plot[200, {x, .5, 1.5},`

`DisplayFunction → Identity], DisplayFunction → $DisplayFunction,`

`PlotLabel → "Antall simuleringer:\t" <> ToString[#1]] & /@ Range[10, nSim, 10];`

Antall simuleringer: 1000