Diversification:
Financial risk vs. demographic risk

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Traditional life insurance/pension undertaking

Contractual payments to insured individuals contingent upon:

- remaining life time (annuity)
- time of death (life insurance)
- occurence and potential duration of disability (long term disability pension)
- dependent's remaining life time (survivor's pension)
- etc.

Risk nature

Risk exposure: Random variations associated with biometric events - "demographic risk"/"biometric risk"

Do away with risk *in the aggregate* by sufficiently large portfolio:

- diversification
- law of large numbers

Assumptions:

- homogenous risks
- independent risks

Funding: Basic principle

Policyholders' obligations in return for insurer's obligation:

· Premium payments

· *In advance*

Pre-funding ⇒ Accumulation of funds

Funding: Technical base

Balance between :

- contractual outgoes
- contractual ingoes and investment income

Balance in *expected* terms and *over time.*

$$E\left(\sum \text{Benefits}\right) = E\left(\sum \text{Premiums} + \text{Return}\right)$$

Principle of equivalence

Carrying out principle of equivalence

Mathematical expectation w.r.t. demographic risk well understood and substantiated from a risk control perspective.

Mathematical expectation w.r.t financial risk:

- what is it?
- how does it work?

Financial risk *not diversifiable*

First attempt to manage financial risk

Pretend that financial risk can be disregarded.

Artificial deterministic discount rate: Sufficiently low to be realised "almost certainly".

Not very satisfactory:

- Theoretically
- In practice

Deterministic discount rate in risky financial market

Setting:

· actual return on insurer's investment *is* stochastic, with some probabilistic properties

· insurer has an accrued liability - represented as a future (stochastic) payment stream

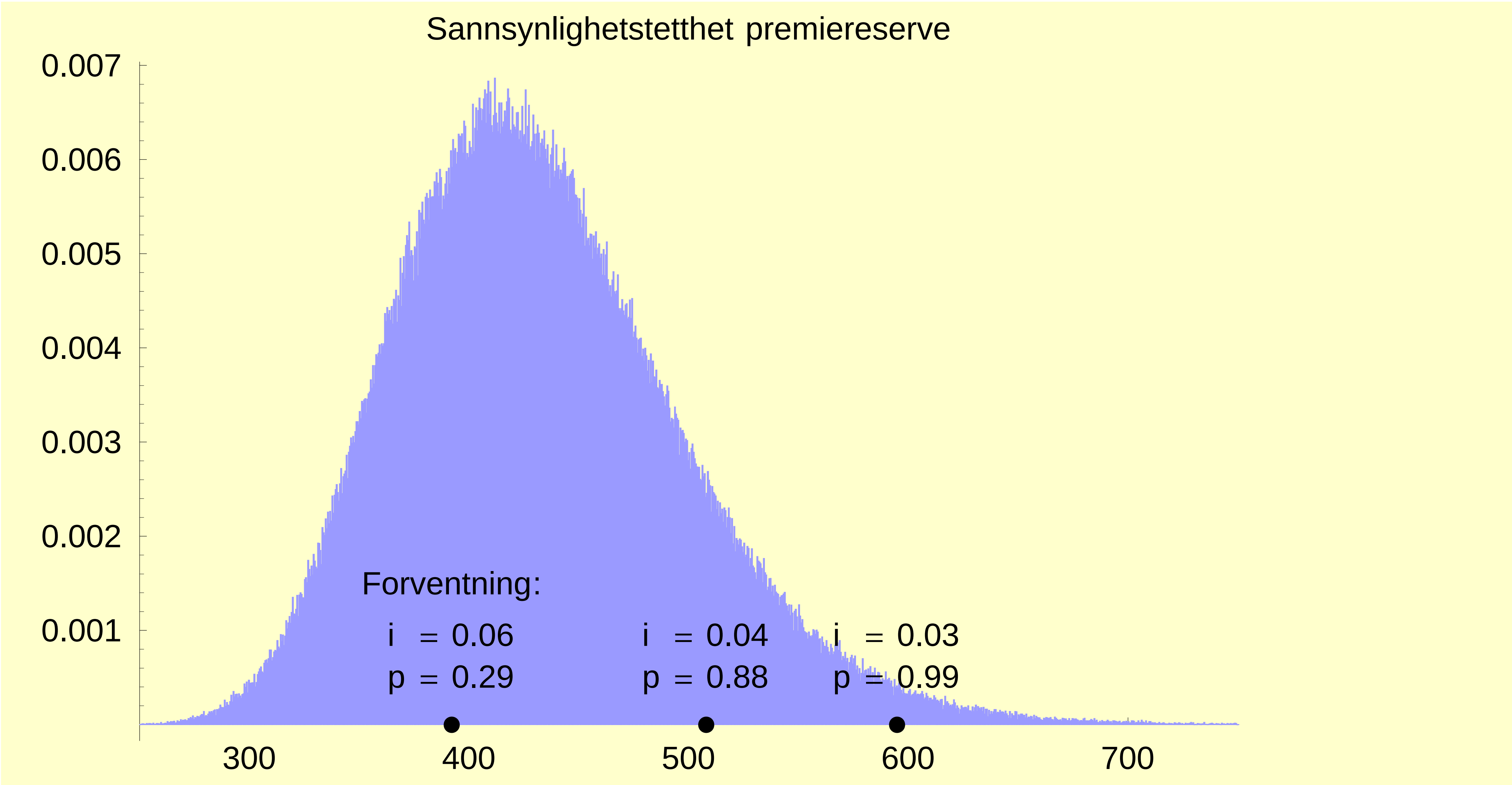
· premium reserve for accrued liability stipulated by discount rate "to the safe side"

Key question: Relation between:

· capital *actually required* to finance insurer's accrued liability - expressed as a probability distribution

· premium reserve - expressed as fixed amount; expected present value as if investment return was deterministic

Grafikk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Case for considering non-diversifiability of financial risk

Actuarial present value of deferred annuity :

$$P = \sum_{t=k}^{\infty} I[T > t] \cdot v_t$$

where

- T = remaining lifetime for insured individual
- v_t = factor for discounting from time t back to time 0.

Non-diversifiability of financial risk: Basis

Two lifes T^1 and T^2 *i.i.d*.

$$P^i = \sum_{t=k}^{\infty} I[T^i > t] \cdot v_t; \; i = 1, 2$$

P^1 and P^2 :

· independent if v_t' s deterministic

· dependent if v_t' s stochastic!

Non-diversifiability of financial risk: Basis

n lifes $T^1, T^2, \ldots, T^n$ $i.i.d.$

$$P^i = \sum_{t=k}^{\infty} I[T^i > t] \cdot v_t; \; i = 1, 2, \ldots, n$$

Assume v_t's stochastic, whereby all P^i's dependent with:
$\text{Var}(P^i) = \sigma^2; \; i = 1, 2, \ldots, n$
$\text{Cov}(P^i, P^j) = \rho \cdot \sigma^2; \; i, j = 1, 2, \ldots, n$

Then:

$$\text{Var}\left(\frac{1}{n} \sum_{i=1}^{n} P^i\right) =$$

$$\frac{1}{n^2} \sum_{i=1}^{n} \text{Var}\,(P^i) + \frac{1}{n} \sum_{i \neq j} \text{Cov}\,(P^i, P^j) = \frac{1}{n^2} \cdot n \cdot \sigma^2 + \frac{1}{n} \cdot n \cdot (n-1) \cdot \rho \cdot \sigma^2 = \sigma^2 \cdot \left[\frac{1}{n} + \left(1 - \frac{1}{n}\right) \cdot \rho\right]$$

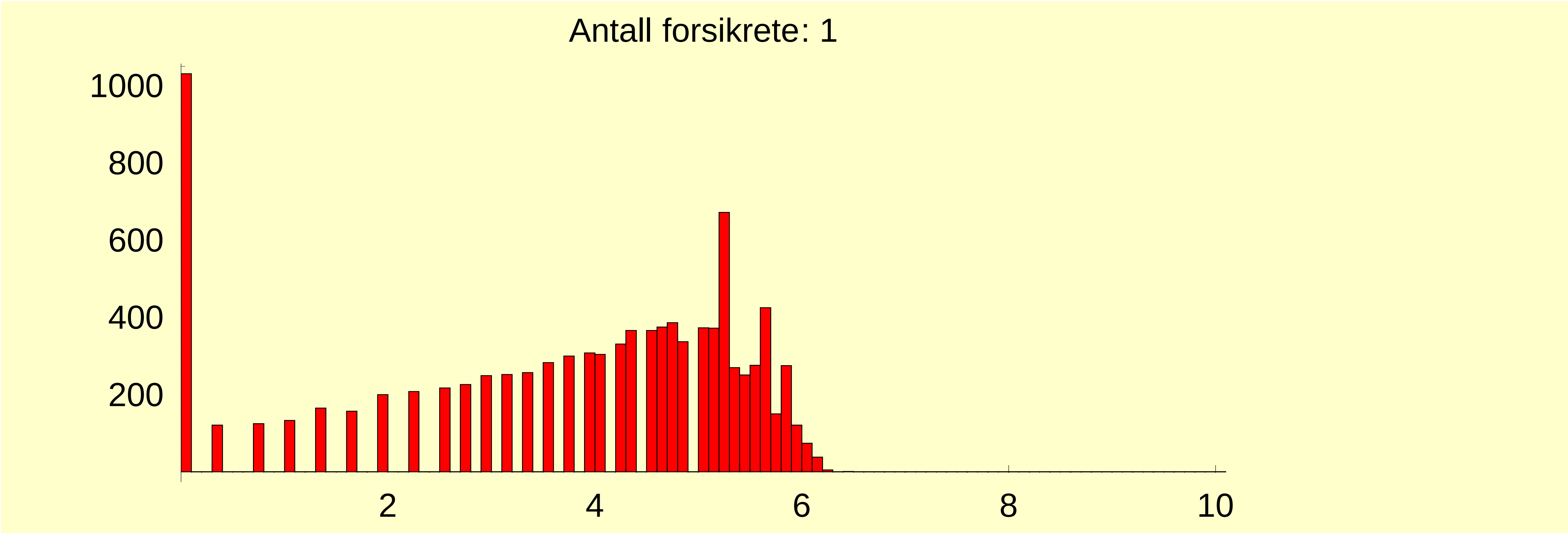
which does not converge to zero as portfolio size increases!

Portofolio uncertainty in the absence of financial risk

Pdf. for:

$\frac{1}{n} \sum_{i=1}^{n} P^i$

under deterministic investment return.

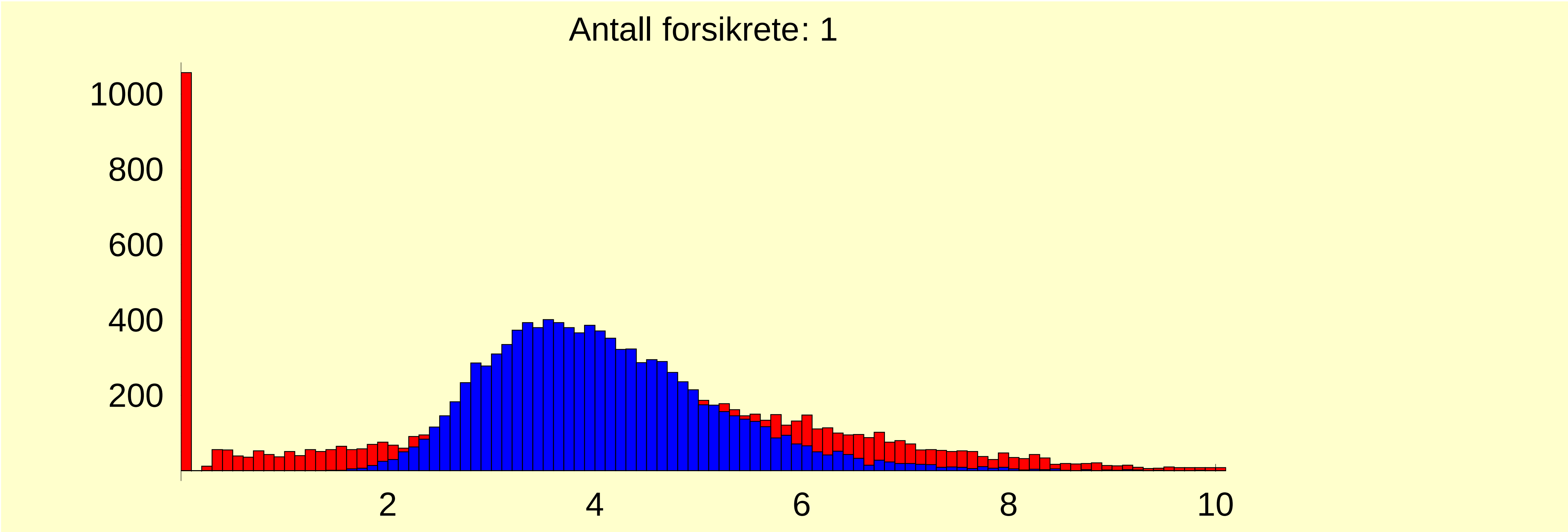


Portofolio uncertainty in the presence of financial risk

Pdf. for:

$\frac{1}{n} \sum_{i=1}^{n} P^i$

under stochastic investment return.



Volatility on investment return → variability of annuity's present value.

Simple financial market model :

$R_t;\ t = 1, 2, \ldots.$ is investment return for period $[t-1,\ t)$

Assume $R_1, R_2, \ldots\ i.i.d.,\ \mu^R = E(R_t)\ \ \sigma^R = \sqrt{\mathrm{Var}\,(R_t)}$
Discount rates :

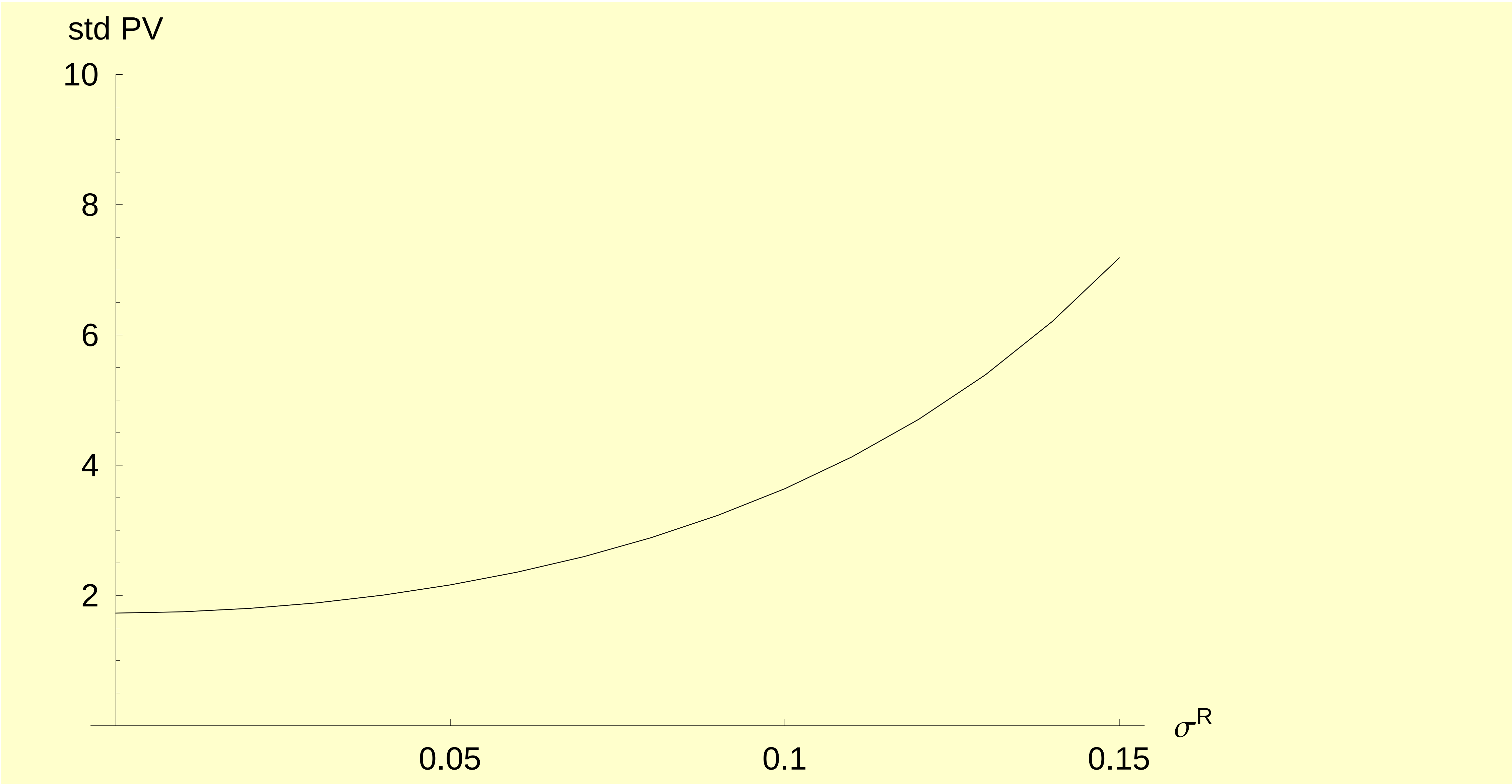
$$v_t = \prod_{s=1}^{t} \frac{1}{1+R_s};\ t = 1,\ 2,\ \ldots$$

Obtain (approximate) probability distribution for

$$P^i\ = \sum_{t=k}^{\infty} I[T^i > t] \cdot v_t$$

by stochastic Monte Carlo simulations of T^i and R_s
realisations. (Has in fact already been done for the preceding graphical illustrations).
In particular : How does increased volatility in portfolio / financial markets
affect the variability of annuity' s present value, as measured by $\mathrm{Std}(P^i;\ \sigma^R)$?.

Standard deviation for annuity's PV depending on standard deviation for investment return.



Correlation between portfolios.

Two portfolios: $\{T^i\}_{i=1}^{n}$ and $\{T'^j\}_{j=1}^{m}$, all $i.i.d$

$$\mathrm{Cov}\left(\sum_{i=1}^{n} P^i,\ \sum_{j=1}^{m} P'^j\right) = \sum_{i,j} \mathrm{Cov}(P^i,\ P'^j) = m \cdot n \cdot \rho \cdot \sigma^2$$

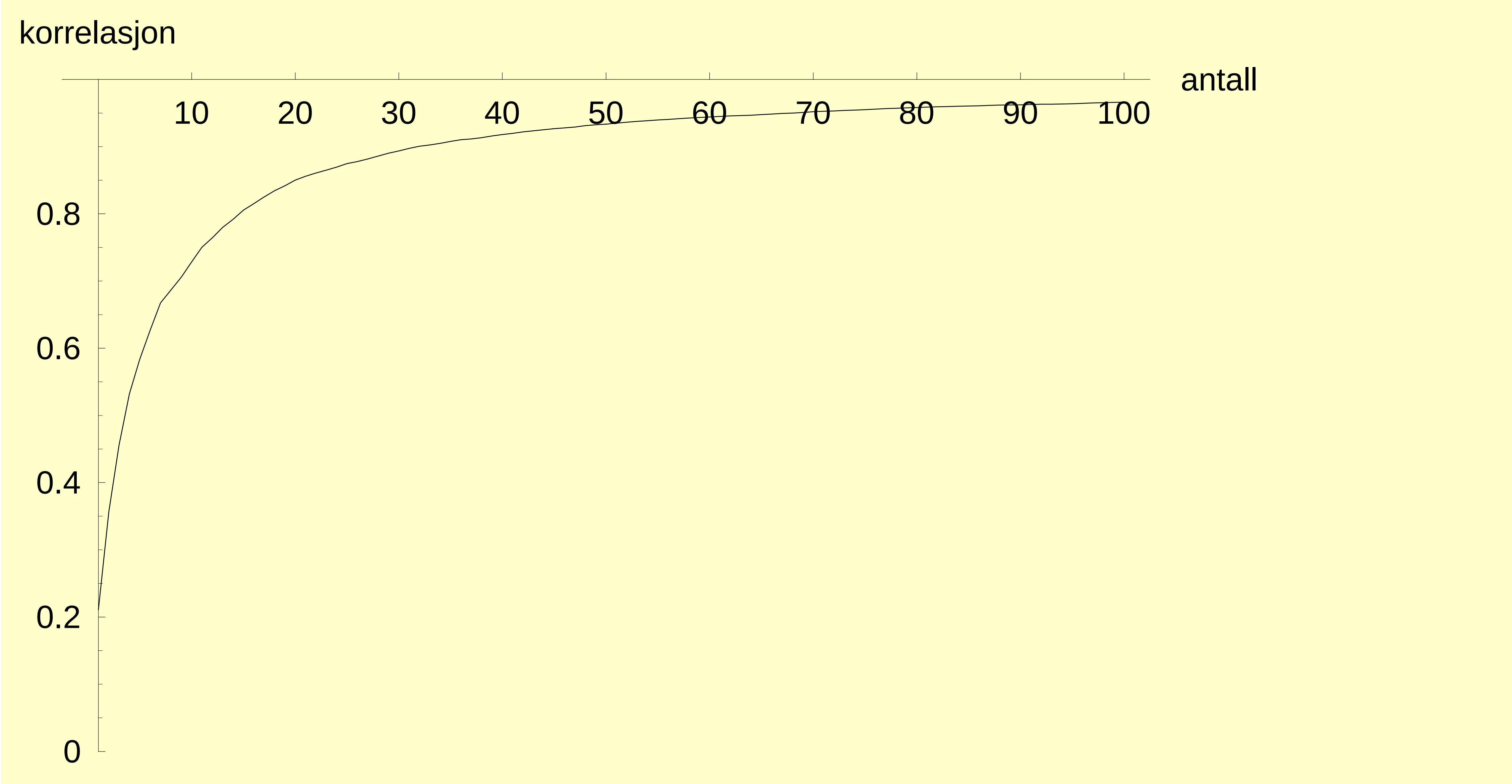
$$\mathrm{Var}\left(\sum_{i=1}^{n} P^i\right) = n \cdot \sigma^2 + n \cdot (n-1) \cdot \rho \cdot \sigma^2$$

$$\mathrm{Var}\left(\sum_{j=1}^{m} P'^j\right) = m \cdot \sigma^2 + m \cdot (m-1) \cdot \rho \cdot \sigma^2$$

$n = m$:

$$\rho\left(\sum_{i=1}^{n} P^i,\ \sum_{j=1}^{n} P'^j\right) = \frac{\rho}{\frac{1}{n} + \frac{(n-1)}{n} \cdot \rho} \xrightarrow{n\to\infty} 1$$

Correlation with increasing portfolio size



Modelling and managing financial risk: Foundation

Financial market model within probabilistic framework:

- (reasonably) realistic
- operational
- possible to validate and estimate against market data/behaviour

Risk issues:

- recall: non-diversifiable
- how to manage?
    - additional capital
    - investment strategies
    - other?
    - possible to completely eliminate the impact of risk?

Financial risk isolated vs. financial risk and liability risk considered as a whole

Course content

Stochastic modelling of financial assets' market value

Pension fund risk in the presence of financial risk

Derivatives:
- Concept
- Pricing
- Hedging

Application: Annual interest rate guarantee for UL/DC contracts

Porfolio theory

Matching of assets to liabilities

Evaluation - Monte Carlo simulation.

Course content - limitations and extended perspective

Basic concepts!

Extended perspective of great importance for life insurance/pensions undertakings:
- Very long time perspective
- Interest rate sensivity
- Possibilities to hedge/securitise long term interest rate guarantees

Regulatory environment:
- Explicit pricing of interest rate guarantee compulsory in Norway from 2007 (2008?)
- Solvency II: Pension and insurance liabilities valued at "market value"
    - expected cash-flows discounted at market interest rates
    - investment strategy to hedge liability risk?