Oppgave 6

Premieutvikling

Parametre

In[1]:=

```
{α, β, c} = {0, 0.0000202, 1.1015};
v = 1/1.03;
x = 30;
n = 35;
ω = 120;
λ = 0.03;
θ = 0.015;
lønn = 500000;
μ = 0.055;
σ = 0.056;
```

Pakker og funksjoner (a = $_{n-t|}\ddot{a}_{x+t}$)

In[11]:=
```
<< "Statistics`ContinuousDistributions`"
```

In[12]:=
```
p[y_, t_] := e^-(α t+ β c^y (c^t-1)/Log[c]);
```

In[13]:=
```
a[t_] := Sum[v^i p[x + t, i], {i, n-t, ω-(x+t)}];
```

Notasjon

simL = simulert lønn på tid t ϵ {0, 1, ..., n-1}

simS = simulert ytelse på tid t ϵ {0, 1, ..., n-1}

simP = simulert premie på tid t ϵ {0, 1, ..., n-1}

simA = simulert avkastning i (t, t+1], t ϵ {0, 1, ..., n-1}

Merk: Alle vektorene er indeksert med +1, f.eks. er **simL[[t+1]]** = L_t

Simuleringsrutine

In[16]:=
```
simSogP := Module[{simL, simS0, simP0},
  simL = FoldList[(1 + λ) #1 + θ #2 #1 &, lønn, RandomArray[NormalDistribution[0, 1], n - 1]]; simS0 = 0.2 simL;
  simP0 = Prepend[Table[((t + 1) simS0[[t + 1]])/n - (t simS0[[t]])/n) a[t], {t, 1, n - 1}], (simS0[[1]] a[0])/n]; {simP0, simS0}]
```

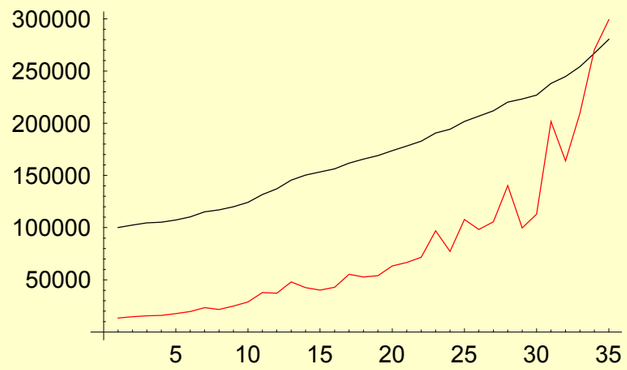
In[17]:=
```
{simP, simS} = simSogP;
```

Simulert bane: Funksjon

In[21]:=
```
visSimulertBane := Show[ListPlot[simS, DisplayFunction → Identity, PlotJoined -> True],
   ListPlot[simP, PlotStyle → RGBColor[1, 0, 0], DisplayFunction → Identity, PlotJoined -> True],
   DisplayFunction → $DisplayFunction, DefaultFont → {"Helvetica", 11}];
```

Simulert bane

In[23]:=
```
visSimulertBane;
```



Avkastning

In[26]:=
```
simA = e^(μ - σ^2/2 + σ RandomArray[NormalDistribution[0,1],n]);
```

In[27]:=
```
simAAlt = Delete[FoldList[e^(μ - σ^2/2 + σ #2) &, 1, RandomArray[NormalDistribution[0, 1], n]], 1];
```

Premiereserve for bestand på tid *t* og forsikringsfond for bestand på tid *t*+1:

In[28]:=
```
premiereserve = Table[{t, ((t + 1) simS[[t + 1]] a[t] p[x, t])/n}, {t, 0, n - 1}];
```

In[29]:=
```
premiereserveRetro = FoldList[#1/v + #2[[1]] p[x, #2[[2]]] &, simP[[1]], Transpose[{Delete[simP, 1], Range[1, n - 1]}]];
```

Alternativ implementering

In[30]:=
```
premiereserveRetroRecur[t_] := premiereserveRetroRecur[t - 1] / v + simP[[t]] p[x, t - 1];
premiereserveRetroRecur[1] = simP[[1]];
premiereserveRetroAlt = Table[premiereserveRetroRecur[t], {t, 1, n}];
```

Testing

In[33]:=
```
premiereserveRetroAlt==premiereserveRetro
```

Out[33]=
```
True
```

In[34]:=
```
premiereserveRetro == Transpose[premiereserve][[2]]
```

Out[34]=
```
True
```

Forsikringsfond

In[35]:=
```
forsikringsfondEksplisittSkrevet = Table[{t + 1, ((t + 1) simS[[t + 1]] a[t] p[x, t] simA[[t + 1]])/n}, {t, 0, n - 1}];
```

In[36]:=
```
forsikringsfond = Table[{t + 1, premiereserve[[t + 1, 2]] simA[[t + 1]]}, {t, 0, n - 1}];
```

Forsikringsfond ved premier etter planen

In[37]:=
```
forsikringsfondVedPremierEtterPlanen =
  FoldList[#1 #2[[3]] + #2[[1]] p[x, #2[[2]]] &, simP[[1]], Transpose[{Delete[simP, 1], Range[1, n - 1], Delete[simA, -1]}]];
```

In[38]:=
```
forsikringsfondVedPremierEtterPlanenRecur[t_] :=
  forsikringsfondVedPremierEtterPlanenRecur[t - 1] simA[[t - 1]] + simP[[t]] p[x, t - 1];
forsikringsfondVedPremierEtterPlanenRecur[1] = simP[[1]];
forsikringsfondVedPremierEtterPlanenAlt = Table[forsikringsfondVedPremierEtterPlanenRecur[t], {t, 1, n}];
```

Testing

In[41]:=
```
forsikringsfondVedPremierEtterPlanenAlt==forsikringsfondVedPremierEtterPlanen
```

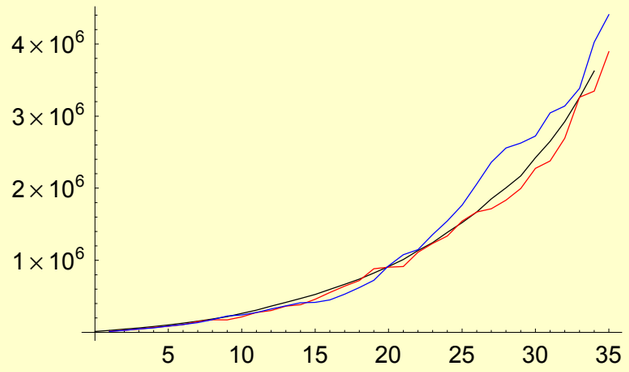
Out[41]=
```
True
```

Testing: Funksjon

In[42]:=
```
visPlott := Show[ListPlot[premiereserve, DisplayFunction → Identity, PlotJoined -> True],
   ListPlot[forsikringsfond, DisplayFunction → Identity, PlotStyle → RGBColor[1, 0, 0], PlotJoined -> True],
   ListPlot[forsikringsfondVedPremierEtterPlanen, DisplayFunction → Identity, PlotStyle → RGBColor[0, 0, 1],
    PlotJoined -> True], DisplayFunction → $DisplayFunction, DefaultFont → {"Helvetica", 11}];
```

Testing

In[44]:=
```
visPlott;
```

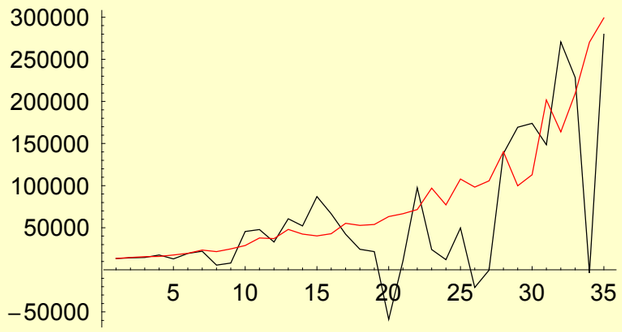


Premier etter planen og faktiske premier som følge av realisert avkastning:

In[46]:=
```
visPlott2 := Show[ListPlot[Prepend[Table[premiereserve[[t + 1, 2]] - forsikringsfond[[t, 2]], {t, 1, n - 1}], simP[[1]]],
    DisplayFunction → Identity, PlotJoined → True],
   ListPlot[simP, PlotStyle → RGBColor[1, 0, 0], DisplayFunction → Identity, PlotJoined → True],
   DisplayFunction → $DisplayFunction, PlotRange → All, DefaultFont → {"Helvetica", 11}];
```

Premier etter planen og faktiske premier som følge av realisert avkastning:

In[47]:=
```
visPlott2;
```



I år med god avkastning, er den faktiske premien negativ!

In[50]:=
```
TableForm[
 Transpose[{Delete[simA, -1], Delete[simP, 1], Table[premiereserve[[t + 1, 2]] - forsikringsfond[[t, 2]], {t, 1, n - 1}]}]]
```

Out[50]//TableForm=

1.05229	14634.2	14327.
1.06368	15593.7	14619.
0.989162	15874.8	17692.1
1.1044	17638.	12978.5
1.032	19620.2	19410.6
1.04168	23424.5	22143.1
1.15186	21593.5	5652.32
1.13738	24835.9	8019.33
0.938671	28801.7	45544.1
0.983272	37768.9	47777.5
1.04478	37104.	32946.5
0.987583	47939.2	60607.5
1.00262	42553.4	52139.
0.917029	40224.7	86999.
0.978827	42932.	66466.
1.05308	55275.3	42347.7
1.07612	52722.5	24355.3
1.07693	53893.2	21647.7
1.19364	63351.4	-58967.3
1.09629	66709.5	10597.3
0.999919	71661.6	97378.4
1.0994	96999.7	24128.1
1.08518	77135.	12075.8
1.07372	107814.	49690.9
1.11332	98187.6	-21051.2
1.09672	105647.	-382.119
1.02738	140299.	138115.
0.989684	99818.1	169362.
0.996338	112995.	173968.
1.04863	201715.	148546.
0.981223	163829.	270643.
1.01679	209506.	228404.
1.11602	270324.	-3908.43
1.02739	299242.	279843.