UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Exam STK4500/STK9500 v2011: Finans og forsikring

Project assignment, disseminated Tuesday 14 June 9.00 hrs., deadline for submission Thursday 16 June 14.00 hrs.. Submission to be delivered in 2 copies at the secretariat ("ekspedisjonen") at the 7th floor.

Together with the written response to the questions in the assignment, the candidate shall attach a printout of the computer-code used for assignment's calculations and printout of the slides to be used at the oral presentation.

The candidate shall confirm that his/hers submission is the result of independent work by attaching a dated and signed declaration.

A supplemental 25 minutes oral presentation will take place 23 and 24 June according to a list which will be published on the course's home page.

For determining the conclusive grade the written solution to the assignment and the oral presentation will be assessed as a whole, with great emphasis being put on the student's ability to explain, elaborate.

In this assignment we consider a financial market which is assumed to comprise 7 financial assets:

1. Norwegian equity
2. Nordic equity
3. International equity
4. Hedgefund
5. Bonds
6. Real estate
7. Money market

We assume that the probabilistic properties of the development of the value for asset i is governed by the following recursion formula:

$$S_t^i = S_{t-1}^i \cdot Exp\left[\left(\mu_i - \frac{\sigma_i^2}{2}\right) + X_t^i\right]; \forall i, X_t \sim N(0, V)$$

with stochastic independence between X_s and X_t for $s \neq t$. The source for the vector μ for the value development's drift and the covariance matrix V for the value development's volatility is the publication "Alternative Investeringer" (2006), published by Orkla Finans.

As part of solving the assignment we shall use the Cholesky decomposition for V, which we will denote by W. μ, V and W are contained in the files "mu.txt", "V.txt" og "W.txt". A printout of these files is also included at the end of the assignment.

Weights for investments in the different financial assets is denoted by $\omega_i; \forall i$.

a) Present an algorithm to generate a simulated pseudo-stochastic realization of the value development for the portfolio over a T year horizon from an initial portfolio value of 1, based on draws of independent standard normal stochastic variables. (Hint: $W^t . W = V$). This algorithm can be used as documentation and description for implementation in a computer program.

For the questions b)-c) let $\omega_i = \frac{1}{7}; \forall i$

b) Find estimates for the upper and lower 2,5% percentile and the 50% percentile for the accumulated value each year, based on simulated realizations of the value development for the portfolio. Illustrate graphically these percentiles over a 40 year horizon and tabulate these percentiles after $1, 3, 5, 10, 20, 30$ and 40 years respectively.

c) Find similar estimates as in b) for the geometric mean of annual returns[1].

d) μ and V are model parameter values. Find the vector of expected annual returns and the matrix of covariances for the annual returns. (Hint: Exercise 2.)

e) Compute the correlation matrix for the annual returns. Comment.

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[1] For annual returns $a_{1,}\ a_2, \dots., a_t$ the geometric mean is defined by: $[\Pi_{s=1}^t (1 + a_s)]^{\frac{1}{t}} - 1$

f) Plot the efficient frontier as a function of expected annual return, when shorting is allowed. Find the efficient set of weights, in the two alternatives where maximum annual standard deviation is 3% and 6% respectively.

We will now consider the present value of an old age/retirement pension in payment, when discounting is done with the returns achieved in the financial market as described in the preceding. We shall use the following intensity for mortality at age $x + t$:

$$\nu_{x+t}(\theta) = (1 - \theta) \cdot \nu_{x+t}(0)$$

$$\nu_{x+t}(0) = \alpha + \beta \cdot c^{x+t}$$

$$\alpha = 0.00007809$$

$$\beta = 0.00000719$$

$$c = 10^{0.04893}$$

θ is a scaling parameter that can be varied to allow for computations with different mortality levels. Additional notation is:

$$T_x^\theta = \text{remaining life time for an individual aged } x\text{, depending on the scaling parameter } \theta$$

$$_t p_x(\theta) = Pr\left[T_x^\theta \geq t\right]$$

g) For age $x = 65$ plot the expected remaining life time, $\bar{e}_x(\theta)$, as a function of θ; $0 \leq \theta \leq 0.3$.

We consider a population of N individuals aged x all having the right to an old age/retirement pension, payable immediately, annually in arrears and for the remainder of life. As an approximation for the actual number of remaining individuals at age $x + t$ we will use $N \cdot_t p_x(\theta)$ (which is assumed to be acceptable when N is large). For convenience we use the scaling $N = 1$ for the computations of the present value of the old age/retirement pension in the remainder of the assignment.

h) For age $x = 65$, $\theta = 0$ and the efficient portfolio where maximum annual standard deviation is 3% find an approximation to the probability distribution for the present value of the pension payments.

i) Compute the expected value of the present value in question h), $\ddot{a}_x(\theta)$, when we generalize to $\theta \neq 0$. Select some values of θ; $0 \leq \theta \leq 0.3$.

j) Compare $\ddot{a}_x(\theta)/\ddot{a}_x(0)$ and $\bar{e}_x(\theta)/\bar{e}_x(0)$ as functions of θ. Comment.

Assume that the situation is as described in question h) and that a medical innovation results in an immediate reduction in the mortality to $\theta = 0.2$. Some may then believe it to be a good idea that the increased cost of providing the old age/retirement pension following the reduction in mortality can be "compensated" by a more offensive investment strategy with a higher expected return.

k) Find (by trial and error) the expected annual return and the standard deviation for the efficient set which results in the same expected present value of the old age/retirement pension after the shift to $\theta = 0.2$ as in question h).

l) Compare the probability distributions of the present value for the two alternatives and comment.

Attachment: Print of the files"mu.txt", "V.txt" og "W.txt":

"mu.txt":

$$\begin{pmatrix} 0.1 \\ 0.1 \\ 0.095 \\ 0.085 \\ 0.08 \\ 0.05 \\ 0.045 \end{pmatrix}$$

"V.txt":

$$\begin{pmatrix} 0.0484 & 0.03146 & 0.0264 & 0.00594 & -0.00264 & 0.000066 & -0.000055 \\ 0.03146 & 0.0484 & 0.0264 & 0.00693 & -0.00176 & 0.00066 & -0.000055 \\ 0.0264 & 0.0264 & 0.0225 & 0.004725 & -0.0012 & 0.00045 & -0.0000375 \\ 0.00594 & 0.00693 & 0.004725 & 0.0081 & -0.00108 & 0.00048 & 0.0000675 \\ -0.00264 & -0.00176 & -0.0012 & -0.00108 & 0.0064 & 0.00012 & 4.\times 10^{-7} \\ 0.000066 & 0.00066 & 0.00045 & 0.00048 & 0.00012 & 0.0009 & 0.000045 \\ -0.000055 & -0.000055 & -0.0000375 & 0.0000675 & 4.\times 10^{-7} & 0.000045 & 0.000025 \end{pmatrix}$$

"W.txt":

$$\begin{pmatrix} 0.22 & 0.143 & 0.12 & 0.027 & -0.012 & 0.0003 & -0.00025 \\ 0. & 0.167186 & 0.0552679 & 0.0183569 & -0.000263181 & 0.00369111 & -0.000115142 \\ 0. & 0. & 0.0710314 & 0.00662319 & 0.00358356 & 0.00295644 & -0.0000159981 \\ 0. & 0. & 0. & 0.0836072 & -0.00926839 & 0.00459963 & 0.00091463 \\ 0. & 0. & 0. & 0. & 0.0784677 & 0.00199583 & 0.0000752432 \\ 0. & 0. & 0. & 0. & 0. & 0.029196 & 0.00141082 \\ 0. & 0. & 0. & 0. & 0. & 0. & 0.00470015 \end{pmatrix}$$