UNIVERSITETET I OSLO
Det matematisk-naturvitenskapelige fakultet

Exam STK4500 v2010: Finans og forsikring

Project assignment, disseminated Tuesday 15 June 9.00 hrs., deadline for submission Tuesday 17 June 14.00 hrs.. Submission to be delivered in 2 copies at the secretariat ("ekspedisjonen") at the 7th floor.

Together with the written response to the questions in the assignment, the candidate shall attach a printout of the computer-code used for assignment's calculations.

The candidate shall confirm that his/hers submission is the result of independent work by attaching a dated and signed declaration.

A supplemental oral examination will take place 24 and 25 June according to a list which will be published on the course's home page.

For determining the conclusive grade the written submission is weighted by 3/4 and the oral examination by1/4.

In this assignment we analyze risk characteristics of defined contribution pension, a modified defined contribution pension with a guaranteed return and an alternative plan which combines basic principles for defined contribution and defined benefit pension.

For all types of pension plans, we consider the same financial market that consists of two financial assets. A unit of each of the assets have values respectively B_0 og S_0 at time 0, and the value of the assets at time t, respectively B_t and S_t , are governed partly by B_t obeying a deterministic geometric growth, while S_t obeys a Geometric Brownian Motion:

$$B_t = B_0 \cdot exp[\mu_B \cdot t]$$

$$S_t = S_0 \cdot exp\left[\left(\mu_S - \frac{\sigma_S^2}{2}\right) \cdot t + \sigma_S \cdot V_t\right]$$

$$V_t \sim N(0, \sqrt{t})$$

Throughout the assignment we shall use the following parameterization for the description of the financial market:

$$\mu_B = 0{,}05$$

$$\mu_S = 0{,}10$$

$$\sigma_S = 0{,}20$$

The asset with value development S_t is called stock and the asset with value development B_t is called bond.

We consider the employer's cost of funding future pension for a single employee. At time 0 the employee becomes a new member of the company pension plan at age $x = 30$ years. The pensionable age is $x + n = 65$ years. The employee's annual salary at time t, L_t, is governed by the following stochastic dynamics:

$$L_0 = 300\ 000$$

$$L_t = (1 + \lambda) \cdot L_{t-1} + \theta \cdot \delta_t \cdot L_{t-1}; t = 1, \dots, n \text{ with } \delta_1, \delta_2, \dots, \delta_n\ i.i.d. \sim N(0,1)$$

$$\lambda = 0{,}03$$

$$\theta = 0{,}015$$

We assume stochastic independence between V_t on the one hand and $\delta_1, \delta_2, \dots, \delta_n$ on the other hand.

For comparison of the cost of financing future retirement benefits, we calculate the present value at time 0 of the contributions/premiums costs from inception of membership in the pension plan to the last premium payment. For this present value calculation, we use the discount rate (annual) $r = 0{,}06$.

We start by considering a defined contribution plan where the annual contribution is paid in advance in annual terms from inception of pension plan membership until the year before reaching the pensionable age at a rate $100 \cdot p^I$% of annual salary at any point in time. Using P_t^I to denote the contribution at time t, we have:

$$P_t^I = p^I \cdot L_t; t = 0, \dots, n-1$$

We will assume that the contribution rate equals 6%, i.e.:

$$p^I = 0{,}06$$

The defined contribution account is credited by investment return achieved in the financial market and mortality inheritance (“dødelighetsarv”). Based on the accumulated pension account at pensionable age an annual pension amount, guaranteed life-long and paid in annual terms, is calculated using a valuation basis[1] with discount rate $i = 0{,}03$ and instantaneous mortality rate (“dødsintensitet”) $\nu_{x+t} = \beta \cdot c^{x+t}$ at age $x + t$, where:

$$\beta = 0{,}0000202$$

$$c = 1{,}1015$$

We assume that the defined contribution account is invested by 20% in stocks and 80% in bonds at any time and that there are no transaction costs of buying and selling these securities.

Spsm. 1. Explain the rebalancing concept that is required to maintain such a fixed relationship 20%/ 80% between stocks and bonds.

Spsm. 2. Using the Monte Carlo simulation approach, find an approximation to the probability distribution of the pension level from the defined contribution plan, measured as the ratio between guaranteed annual pension benefit and annual salary at reaching the pensionable age, and illustrate the approximate probability distribution graphically. What is the expectation and standard deviation of the approximate probability distribution?

Spsm. 3. Using the Monte Carlo simulation approach, find an approximation to the probability distribution for the present value of the annual contributions, and illustrate the approximate probability distribution graphically. What is the expectation and standard deviation of the approximate probability distribution?

We now modify the defined contribution plan by introducing an investment return guarantee: return credited to the defined contribution account shall be the greater of the returns achieved in the financial market and a guaranteed annual return r^g. We use the term defined contribution plan with guaranteed return for this plan.

We will now study the pension levels and pension costs of defined contribution plan with guaranteed return, and compare with the defined contribution plan with no guaranteed return. For the alternative with guaranteed return *the annual contribution rate is adjusted so that the expected value of the pension level is the same as with no guaranteed return.*

Spsm. 4. Explain why the same underlying stochastic scenarios for the financial market development and salary development should be used in the comparison.

Spsm. 5. For the three alternatives $r^g = 0{,}00, r^g = 0{,}015$ and $r^g = 0{,}03$ use Monte Carlo simulation to obtain an approximation to the probability distributions for the pension level of the defined contribution plan with guaranteed return, and illustrate the approximate probability

[1] With a discount rate $i = 0{,}03$ and the financial market and asset allocation as described, there will be “investment surplus” during the course of the pension payment, which could rise to an adjustment to the guaranteed pension amount. Since the focus for this assignment is the impact of the stochastics before the pensionable age, we disregard any such pension adjustment.

distributions graphically. What are the adjusted contribution rates for the three alternatives, and what is the standard deviation of the pension level. Comment.

For hedging and financing the guaranteed return, we now assume that an annual premium is paid in advance to the pension plan provider and that the provider assumes the responsibility to meet the investment guarantee with final effect. The price for the guaranteed return is determined using Black Scholes.

Spsm. 6. What is the unit price for the investment guarantee for the three alternatives $r^g = 0{,}00$, $r^g = 0{,}015$ and $r^g = 0{,}03$?

Spsm. 7. Using the Monte Carlo simulation approach, find an approximation to the probability distribution for the present value of the total annual contribution/premium payments for the three alternatives $r^g = 0{,}00$, $r^g = 0{,}015$ and $r^g = 0{,}03$, and illustrate the approximate probability distributions graphically. What is the expectation and standard deviation of the approximate probability distributions? Compare with expectation and standard deviation in **Spsm. 3** and comment.

An alternative for financing the investment guarantee is that insufficient return in the financial market compared to the guarantee is covered by the employer.

Spsm. 8. Using the Monte Carlo simulation approach, find an approximation to a probability distribution for the present value of the total annual contribution/premium payments for the three alternatives $r^g = 0{,}00$, $r^g = 0{,}015$ and $r^g = 0{,}03$, and illustrate the approximate probability distributions graphically. What is the expectation and standard deviation of the approximate probability distribution? Compare with the results of **Spsm. 7** and comment (Help for comparison with **Spsm. 7** is that the Black Scholes price of the investment guarantee with $r^g = 0{,}015$ is $0{,}003341$).

We now introduce an alternative concept for adjusting the value of the pension account, where the movement in the account value from time t (after the account has been credited with the contribution at this time) to time $(t+1)$ is $Max\left[f_1 \cdot \left(0{,}2 \cdot \frac{S_{t+1}-S_t}{S_t} + 0{,}8 \cdot \frac{B_{t+1}-B_t}{B_t}\right), f_2 \cdot \frac{L_{t+1}-L_t}{L_t}\right]$. f_1 and f_2 are parameters to be fixed for concrete applications.

Spsm. 9. How would you verbally describe this concept for adjusting the pension account? (Hint: What do the parameter values $\{f_1 = 1, f_2 = 0\}$ and $\{f_1 = 0, f_2 = 1\}$ correspond to?)

Spsm. 10. Assume that $f_2 = 1$. Using the Monte Carlo simulation approach, determine the (approximate) value of f_1 which results in the same expected pension level as with the defined contribution plan without guarantee, when we use the same contribution rate ($p^I = 0{,}06$).

We assume that the mechanism for securing and funding the adjustment of the account is such that the difference between returns in the financial market and the adjustment cost is borne by the employer (and that the employer will be credited the difference if the achieved return is higher than the adjustment cost).

Spsm. 11. For the alternative $f_2 = 1$ and f_1 obtained in **Spsm. 10** use the Monte Carlo simulation approach to obtain an approximation to the probability distribution for the present value of the total annual contribution/premium payments and illustrate the approximate probability distribution graphically. What is the expectation and standard deviation of the approximate probability distribution? Compare with the results of **Spsm. 8** and comment.