

## Exercises STK4500

### Exercise E1

Suppose an insurance arrangement defines payments  $s_l(i)$  which depends on the state  $C_l = i$  of an individual when she or he is in age  $l$ . The contract is set up at time  $k = 0$  when the policy holder is in age  $l_0$  and state 1 and lasts  $K$  periods with all payments in advance so that the last of them takes place at  $k = K - 1$ . Introduce the necessary mathematical quantities characterizing the state process  $C_l$  yourself, in particular the transition probabilities  $p_l(i|j) = \Pr(C_{l+1} = i|C_l = j)$  which are assumed known.

- a) How do you compute the key quantities  $\Pr(C_l = i|C_{l_0} = 1)$  under a standard assumption?
- b) Define the present value of the entire scheme at  $k = 0$  when the discounts are based on a fixed technical rate of interest  $r$ .
- c) How is a) modified when the valuation is of the fair value type based on market discounting?
- d) Write a short essay (at most ten lines) on the sources of uncertainty in a) and b).
- e) If  $c = 1$  is the state where the policy holder makes contributions to the scheme, write down an equation defining the equivalence premium.
- f) How do you define the value of the scheme at time  $k$ ?
- g) Consider a disability scheme with an active and a disabled state that terminates at  $K = l_r - l_0$  where  $l_r$  is the retirement age. Rehabilitation is included. Explain how the mathematical set-up in this exercise can be used to compute the equivalence premium and the value at time  $k$ .

### Exercise E2

Consider a pension scheme where the individual is in age  $l_0$  when the scheme is set up at time  $k = 0$ . Premia are paid in advance until retirement age  $l_r$  and a pension  $s$  is then received at the start of each period until the policy holder dies. Discounting is by means of a fixed, technical rate of interest  $r$ . Suppose first that the insurer pockets the money in case of an early death before the retirement age is reached.

- a) Define the concept of a life table and explain how it is related to the probabilities  $q_l$  of an individual of age  $l$  dying the coming period.
- b) Use the life table to write down a mathematical expression for the value of all the premium payments at time  $k = 0$ .
- c) The same question for the value of the pension.
- d) What is the equivalence premium of the scheme?

e) If the policy holder decides to sell the value of the contract back to the insurer, what would the price be?

f) Suppose the entire pension had been paid for at  $k = 0$  when the arrangement was set up. What is the equivalence premium now?

### Exercise E3

Suppose the policy holder in Exercise E2 believes that her/his consumption at very old age will be lower than in earlier years and decides on a strategy where the pension is changed from  $s$  to  $s' < s$  after age  $l'_r > l_r$ .

a) What is the equivalence premium now?

b) And what is the value of the contract at time  $k$ ?

### Exercise E4

The situation is the same as in Exercise E2 except that we now consider term insurance where the insurer pays out a sum  $s_1$  if the policy holder dies before age  $l_1 = l_0 + K$ . A premium  $\pi$  is contributed in the beginning of each of the periods the contract lasts.

a) Define the probabilities  ${}_kq_{l_0}$  needed to value the contract and explain how they are related to the life table in Exercise E2. b) Write down a mathematical expression for the equivalence premium.

c) What is the value of the contract at time  $k$ ?

d) Suppose the holder decides to break off the arrangement at time  $k$ . Offer an intuitive explanation for why she or he would be entitled to some payback from the company.

### Exercise E5

Suppose the contract in Exercise E4 is modified so that the beneficiary following the death of the policy holder instead of a one-time sum receives a pension  $s$  while alive. To fix things consider a classical widow scheme where the policy holder is male and the beneficiary female. If she dies first, the insurance company takes over the account that has been built up. Premia are paid in the beginning of each of the first  $K$  periods. To answer the questions below you will have to introduce some reasonable additional assumptions concerning the deaths of the two partners.

a) Why do you in practice need two sequences of mortalities, say  $q_l$  and  $q'_l$  to value such arrangements?

b) Write down a mathematical expression for the present value of the premia at the time  $k = 0$  when the arrangement is set up.

c) The same question for the pension received by the widow so that you can write down the equivalence premium.

d) What is the value of the contract at time  $k$ ?

e) Suppose the contract in Exercise E5 is modified so that there is in addition to the widow pension a one-time payment  $s'$  to the couple at time  $K$  if both are alive at that time. Write down an expression for the equivalence premium now.