UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: STK4500/9500 — Finance and Insurance

Day of examination: Tuesday June 11'th 2013

Examination hours: 09.00 – 13.00

This problem set consists of 3 pages.

Appendices: None

Permitted aids: Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

Consider single life arrangements with p_l the probability that an individual at age l survives the coming period. There is a fixed technical rate of interest r.

a) What is a life table $_kp_l$ and how it is computed from the p_l's?

b) Use the life table to calculate the present value of a pension v starting at age l_r with l_0 the age when the contract is set up.

c) What is the equivalence premium when the entire pension is paid for in the beginning?

d) Write down a mathematical expression for the probabilities $_kq_l$ you need to value term insurance contracts where beneficiaries receive a sum s upon the death of the policy holder.

e) If a term insurance contract lasts K periods with premium payments in equal amounts at the start of each period, what is the equivalence premium for a policy holder of age l_0 intially?

Problem 2

Mutual term insurance is a common type of contract where the surviving member of a couple receives a benefit s upon the death of the other. Assume below that the couple is male/female and lasts K periods. A premium π is contributed at the start of each period, and there is no remuneration if both are alive when the contract is terminated. Discounting is as in Problem 1. You have to introduce the necessary additional quantities and assumptions yourself.

a) Write down a reasonable mathematical expression for the present value of the premia at the time the contract is agreed.

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b) The same question for the present value of the benefit.

c) How do you determine the equivalence premium?

d) What is the value of the contract at time k when both individuals are alive? Explain why it is likely to be positive so that the couple is entitled to compensation if they decide to break it off.

e) If the couple wants a non-symmetrical arrangement with benefit s_i when individual i dies for $i = 1, 2$, how are the preceding calculations affected?

Problem 3

Let $\mathcal{X}_0, \ldots, \mathcal{X}_K$ be a sequence of given liabilities so that the present value under the discount scheme $d_0, \ldots, d_k$ is

$$\mathcal{PV} = \sum_{k=0}^{K} d_k \mathcal{X}_k.$$

The net value of the portfolio is then $\mathcal{B} = \mathcal{V} - \mathcal{PV}$ where $\mathcal{V}$ is the value of the assets held. Assume everything to have been invested in bonds which releases an amount B_j at time j for $j = 0, \ldots, J$. The book value $\mathcal{B}$ is a relevant measure of the soundness of the portfolio. Envisage a large K, say 50 years or more.

a) What is d_k under the fixed rate of interest r in Problems 1 and 2?

b) Explain what is meant by market discounting (also known as fair value) and supply two versions of such schemes.

c) Why are the two schemes in b) likely to produce the same result?

d) Explain how the bond portfolio can be designed to reduce the uncertainty in $\mathcal{B}$ under market discounting and why it is advantageous from this point of view that J is as large as possible.

Problem 4

Let $R_1, \ldots, R_K$ be equity returns in K consecutive periods oscillating according to

$$\log(1 + R_k) = \xi + \sigma\varepsilon_k, \qquad k = 1, \ldots, K$$

with ξ and σ fixed parameters and with $\varepsilon_1, \ldots, \varepsilon_K$ independent and normal$(0, 1)$.

a) Argue that the accumulated return $R_{0:K}$ over all K periods can be specified as

$$\log(1 + R_{0:K}) = K\xi + \sqrt{K}\sigma\varepsilon$$

where ε is normal$(0, 1)$.

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b) Find mathematical expressions for $E(R_{0:K})$ and $\mathrm{sd}(R_{0:K})$ utilizing that if ε is normal$(0, 1)$, then

$$E(e^{\xi+\sigma\varepsilon}) = e^{\xi+\sigma^2/2} \qquad \text{and} \qquad \mathrm{sd}(e^{\xi+\sigma\varepsilon}) = e^{\xi+\sigma^2/2}\sqrt{e^{\sigma^2}-1}.$$

Suppose the model is extended so that

$$\log(1 + R_k) = \xi + \sigma_k\varepsilon_k,$$

where

$$\sigma_k^2 = (1 - \theta_1 - \theta_2)\xi_\sigma^2 + \theta_1(\log(1 + R_{k-1}) - \xi)^2 + \theta_2\sigma_{k-1}^2.$$

This applies for $k = 1, \dots, K$ with ξ_σ, θ_1 and θ_2 parameters. In the beginning $\sigma_0 = \xi_\sigma$.

c) Write a Monte Carlo program which enables you to approximate the distribution of $R_{0:K}$.

d) Explain how you use the program to compute the mean and standard deviation of $R_{0:K}$.

e) A check of the program when $\xi = 0.04$, $\xi_\sigma = 0.25$, $\theta_1 = 0$, $\theta_2 = 0$ and $K = 10$ with 1000000 simulations, gave $E(R_{0:K}) \approx 1.043$ and $\mathrm{sd}(R_{0:K}) \approx 1.901$. Argue that this suggests that the program is correct.

f) The table below shows the percentiles of the return after ten years both under the conditions in e) with volatilities fixed (second row) and when they are stochastic with $\theta_1 = 0.2$ and $\theta_2 = 0.6$ (third row). Again 1000000 simulations were used. Judge the impact of stochastic volatilities of this magnitude on the uncertainty of ten-year equity returns.

Percentiles	0.01	0.05	0.25	0.50	0.75	0.95	0.99
$\theta_1 = 0$, $\theta_2 = 0$	-0.76	-0.59	-0.13	0.49	1.54	4.48	8.37
$\theta_1 = 0.2$, $\theta_2 = 0.6$	-0.79	-0.58	-0.09	0.49	1.45	4.38	9.50

END