Obligatory assignment for STK4500, Spring 2018

You are to write a report where you clearly describe the computations you have made, defining all symbols you use, and interpret the results you get. It may be useful to include the code that you have used for the computations. That is however not a substitution for an explanation in words. The report must be handed in **electronically** within **April 16** in the Devilry system (see here).

The assignment consists in computing the Solvency Capital Requirement (SCR) of a life insurance company. This consists of several components. To pass, you must have made a serious attempt at finding all of them, as well as combining them into a final result.

Cooperation with others is not forbidden, but the report itself must be written by you **alone**.

Some R-code that you may need is given in the file `oblig_2018.R`

Objective

The objective in this assignment is to compute the Solvency Capital Requirement (SCR) for a life insurance company, using the standard model presented in the note on Solvency II. You may ignore operational risk and the adjustment term, so that the SCR is simply equal to the Basic Solvency Capital Requirement (BSCR). The BSCR is again computed based on the SCR for liability and market risk. The correlation between SCR^{liab} and SCR^{mark}, needed to aggregate them into $BSCR$ is to be found in the note on Solvency II. You should compute the $BSCR$, but first, you must compute the components SCR^{liab} and SCR^{mark}.

Liability risk

Here, you only need to consider mortality and longevity risk, and may ignore the other five types of liability risk. The mortality risk is linked to a term insurance portfolio, whereas the longevity risk is related to a pension insurance portfolio. These er sketched below. The correlation between SCR^{mort} and SCR^{long}, needed to aggregate them into SCR^{liab} is to be found in the note on Solvency II. Further, you may assume that there is no reinsurance on the two portfolios, such that the SCR's are given by

$$\sum_{k=1}^{K} \frac{L_k^S}{(1+r_k)^k} - \sum_{k=1}^{K} \frac{L_k}{(1+r_k)^k},$$

where L_k^S and L_k are the liabilities under stressed and normal conditions, respectively, and r_k is the market risk free interest rate at time k, the model for which is given below. You should the SCR^{liab}, but first you must compute the components SCR^{mort} and SCR^{long}.

Portfolio 1: term insurance

This is insurance against death with a one-time payment $s_1 = 2$ transferred a beneficiary upon death of the policy holder. All the contracts are equal. They were agreed at $l_0 = 30$ years and lasts $K = 25$ years with premia π_1 paid in advance and the benefit (if there is any) is transferred immediately after death has occurred (which means at the end of the current year).

Argue that the expected present value of this contract is

$$\mathcal{PV}_0 = -\pi_1 \sum_{k=0}^{K-1} d^k \ {}_k p_{l_0} + s_1 \sum_{k=1}^{K} d^k \ {}_k q_{l_0},$$

where the discount d is based on the technical rate $r = 2\%$. The survival probabilities follow the Gompertz-Makeham model given below. Now, compute π_1 under equivalence. Further, explain why the liabilities $L_{11}, \dots, L_{1K}$, needed to compute SCR^{mort}, are given by

$$L_k = -\pi_1 \sum_{m=0}^{K-k} N_{1m} \ {}_k p_{l_0+m} + s_1 \sum_{m=0}^{K-k} N_{1m} \ {}_k q_{l_0+m}, \ \ k = 1, \dots, K-1,$$

and

$$L_K = s_1 N_{10} \ {}_K q_{l_0},$$

where N_{1k}, $k = 0, \dots, 25$, is the number of policy holders starting the contract k years ago. The age distribution is given below. Based on this, compute SCR^{mort}.

Portfolio 2: pension

This is a pension portfolio of equal policies. All the contracts were drawn up at the age of $l_0 = 30$ years and last to the end of life which you define as $l_e = 120$. The retirement age is $l_r = 67$ years, and the annual pension received after that date is $s_2 = 0.3$. Prior to retirement a premium π_2 is contributed in advance.

Argue that the expected present value of such a contract is

$$\mathcal{PV}_0 = -\pi_2 \sum_{k=0}^{l_r-l_0-1} d^k \ {}_kp_{l_0} + s_2 \sum_{k=l_r-l_0}^{l_e-l_0} d^k \ {}_kp_{l_0}.$$

The discount and the survival probabilities are as for the term insurance portfolio. Now, compute π_2 under equivalence. Further, explain why the liabilities $L_{21}, \ldots, L_{2,l_e-l_0}$, needed to compute SCR^{mort}, are given by

$$L_k = -\pi_2 \sum_{m=0}^{l_r-l_0-1-k} N_{2m} \ {}_kp_{l_0+m} + s_2 \sum_{m=l_r-l_0-k}^{\min(l_e-l_0-k,65)} N_{2m} \ {}_kp_{l_0+m}, \ k = 1, \ldots, l_r-l_0-1,$$

and

$$L_k = s_2 \sum_{m=0}^{l_e-l_0-k} N_{2m} \ {}_kp_{l_0+m}, k = l_r - l_0, \ldots, l_e - l_0,$$

where N_{2k}, $k = 0, \ldots, 65$, is the number of policy holders starting the contract k years ago. The age distribution is given below. Based on this, compute SCR^{long}.

Life table

The survival probabilities are given by the Gompertz-Makeham model

$$p_l = e^{-\theta_0-(\theta_1/\theta_2)(e^{\theta_2}-1)e^{\theta_2 l}},$$

with

$$\theta_0 = 0.00078, \ \theta_1 = 0.0000376, \ \theta_2 = 0.092759.$$

Age distribution

The number N_{1k} of individuals in Portfolio 1 who set up their contract k years earlier is given by

$$N_{1k} = C_1 e^{-\gamma_1 |k-\mu_1|},\ k = 0, \dots, 25,$$

with

$$C_1 = \frac{J_1}{\sum_{k=0}^{25} e^{-\gamma_1 |k-\mu_1|}},\ \gamma_1 = 0.10,\ \mu_1 = 20,$$

$J_1 = 150,000$ being the total number of individuals. The number N_{2k} of individuals in Portfolio 2 who set up their contract k years earlier is given by

$$N_{2k} = C_2 e^{-\gamma_2 |k-\mu_2|},\ k = 0, \dots, 65.$$

with

$$C_2 = \frac{J_2}{\sum_{k=0}^{65} e^{-\gamma_2 |k-\mu_2|}},\ \gamma_2 = 0.15,\ \mu_2 = 24,$$

$J_2 = 200,000$ being the total number of individuals.

Risk free interest rate

The market risk free interest rate is given by

$$r_k = a + b(1 - e^{-\theta k}),$$

where

$$a = 0.01,\ b = 0.15,\ \theta = 0.1.$$

Market risk

You may ignore currency and concentration risk. This means that the market risk is divided into interest rate, equity, property and spread risk. The correlations between the corresponding SCR's are given in the note on Solvency II, letting ρ be 0.5 since you will consider the risk of decreasing interest rates. Further, the total assets A of the company are $300,000$, of which 30% is invested in equity, 10% in property and 60% in bonds. You should the SCR^{mark}, but first you must compute the components SCR^{int}, SCR^{equi}, SCR^{prop} and SCR^{spread}.

Interest rate

SCR^{int} is given by $BE^S - BE$, where $BE = BE_1 + BE_2$ from the two portfolios under the liability risk, and BE^S is computed as BE, but with a shocked interest rate curve. The shocks for the interest rate curve are given in the note on Solvency II for some time steps. To get the remaining ones, you need to interpolate. Linear interpolation may be done with the R-command `approx` with `rule=2`. For instance, if one has observed $S_k = (0.31, 0.29, 0.27)$ for $k = 10, 12, 15$, and you want the values for $k = 10, \ldots, 20$, then you may use the command

```
interp <- approx(c(10,12,15),c(0.31,0.29,0.27),xout=10:20,rule=2).
```

The corresponding interpolated values are then given by `interp$y`. Now, compute SCR^{int} under the standard model.

Equity

The stock investments are spread on 20% in strategic, or Type 1, and 80% in not strategic, or Type 2, stocks. Further, you may assume that these investments are not protected by options. Now, compute SCR^{equi} under the standard model.

Property

Compute SCR^{prop} under the standard model.

Spread

The bond investment is divided $50 - 50$ between two bonds, one with AA rating and duration 5.5 and one with AAA rating and duration 10.5. Now, compute SCR^{spread} under the standard model.