# STK4500: Life Insurance and Finance

Exercise list 2

## Exercise 2.1 (Mortality basis from The Financial Supervisory Authority of Norway)

Finanstilsynet (The Financial Supervisory Authority of Norway) published the following letter in 2013 with the mortality basis Norwegian insurances companies have to comply with. The document is popularly known as $K2013$. Go to K2013 to download it. There you will find different functions for $\mu(x,t)$ according to gender and risk. Here, $x$ is the age of the insured and $t$ is the calendar year. As you know, mortality changes from year to year in the sense that, a person who is $x$ today, say $t = 2021$ will not have the same mortality as a person who is $x$ years old next year $t = 2022$.

(a) Consider two states $S = \{*, \dagger\}$ and $\mu(t) = \mu_{*\dagger}(t)$, $t \geqslant 0$ the mortality rate. Use Kolmogorov's equation to show that the probability of an $x$ year-old will survive $s$ more units of time, given that they are alive at $x+t$ is given by

$$p_{**}(x+t, x+s) = \exp\left(-\int_t^s \mu(x+u)du\right), \quad s,t \geqslant 0, \quad s \geqslant t.$$

(b) If we take $\mu(t) = \mu_{Kol}(x,t)$ where $\mu_{Kol}$ are the mortality rates from Finanstilsynet, then for a life aged $x$ in year $Y \geqslant 2013$ we have

$$p_{**}(x+t, x+s) = \exp\left(-\int_t^s \mu_{Kol}(x+u, Y+u)du\right), \quad s,t \geqslant 0, \quad s \geqslant t. \tag{0.1}$$

In particular, the probability of surviving one more year given that one is $x$ years old in 2021 is given by

$$p_{**}(x, x+1) = \exp\left(-\int_0^1 \mu_{Kol}(x+u, 2021+u)du\right).$$

Use Taylor's formula (of order one) to prove the (rather very rough) approximation

$$p_{**}(x, x+1) \approx \exp\left(-\mu_{Kol}(x, 2021)\right).$$

(c) In general, we prefer a more accurate integration method to find $p_{**}$. Use Riemann sums, trapezoidal rule and Simpson's method for finding an approximate value for

$$\int_a^b f(t)dt,$$

for a Riemann integrable function $f$ on $[a, b]$.

Apply this to $K2013$ with mortality risk and compute

$$p_{**}(x, x+t), \quad t \in \{0, 10, 20, 30, 40, 50\},$$

where $x$ is your age.

Plot the function $t \mapsto p_{**}(x, x+t)$ where $x$ is your age. Remember that you have to choose $\mu_{Kol}$ according to your gender.

(d) Write an R-code which generates random lives with the mortalities given by Finanstilsynet. Plot many life times in a histogram. Compute the empirical descriptive statistics and check that they are close to the theoretical ones.

Hint: Fix $x$ and let $T$ be the remaning life time of an $x$ year old person. Then the total life time is $T_x \triangleq x + T$. What is the distribution (function) of $T_x$?. When you detect the distribution function of $T_x$, use the inverse transform sampling method to simulate values from $T_x$. The inverse transform method is based on the following result: if $Z$ is a random variable with distribution function $F_Z$ then $F_Z(Z)$ is uniformly distributed on $[0, 1]$.

## Exercise 2.2 (Disability income insurance)

A disability income insurance provides benefits during periods of sickness. There are no benefits after recovery. Typically, such a policy provides an annuity while the person is sick, whereas premiums are paid while the insured is healthy. On the other hand, it could be also used for valuing lump sum payments in the case of sickness of death.

Assume in this model that the state of the insured $X_t \in S$ is described by a regular Markov chain with state space $S = \{*, \diamond, \dagger\}$ where $*$ ="healthy", $\diamond$ ="sick" and $\dagger$ ="dead". Suppose that the transition rates are given by the Gompertz-Makeham model as follows

$$\mu_{*\diamond}(t) = a_1 + b_1 \exp(c_1 t) \quad \mu_{\diamond *}(t) = 0.1\mu_{*\diamond}(t), \quad \mu_{*\dagger}(t) = a_2 + b_2 \exp(c_2 t) \quad \mu_{\diamond\dagger}(t) = \mu_{*\dagger}(t),$$

where $a_1 = 4 \cdot 10^{-4}$, $b_1 = 3.4674 \cdot 10^{-6}$, $c_1 = 0.138155$, $a_2 = 5 \cdot 10^{-4}$, $b_2 = 7.5858 \cdot 10^{-5}$ and $c_2 = 0.087498$. Compute $p_{**}(x, x+10)$ and $p_{*\diamond}(x, x+10)$ for $x = 60$ (years). Simulate and draw the graphs of each transition probability for different values of $x$, say $x \in [0, 100]$.

## Exercise 2.3

Assume an endowment policy with 250 000\$ death benefit and 125 000\$ survival benefit. The yearly premium (in continuous time) is given by 2 200\$. Suppose that the insured is 25 years at the beginning of the contract, which expires at the age of 65. Further, death occurs at 45 years. Let $\delta(s) = \log(1 + 0.03)$. Compute the prospective reserve $V^+(t, A)$ for $t = 30$ years.