STK4500: Life Insurance and Finance

Exercise list 3

Exercise 3.1 (Interest rates in a periodic economy)

Consider interest rates in a "periodic economy" with a period of 8 states. Let us assume that such interest rates are modeled by a homogeneous Markov chain i_t, $t = 0, 1, 2, \ldots$, with the following transition probabilities for the corresponding states:

State	Interest rate (%)	p_{ii}	p_{ii+1}	p_{ii+2}	
0	Start	5.0	0.1	0.7	0.2
1	Increasing interest	5.5	0.1	0.7	0.2
2	Max. interest	6.0	0.1	0.7	0.2
3	Decreasing interest	5.5	0.1	0.7	0.2
4	Average	5.0	0.1	0.7	0.2
5	Decreasing interest	4.5	0.1	0.7	0.2
6	Min. interest	4.0	0.1	0.7	0.2
7	Increasing interest	4.5	0.1	0.7	0.2

Calculate the probability that i_t is in state 2, given that i_0 is in state zero for $t = 4$.

Exercise 3.2

Consider a disability insurance. Suppose for this model that there is no waiting period and that the disability pension is given by the fixed amount of 20 000\$ per year until the age of 65 years. Assume that annual premiums are 2 500\$ until the age of 65 years and that the age of the insured at the beginning of the contract is $x = 30$ years.

Determine the policy functions $a_i(t)$ (generalized pension payments) and $a_{ij}(t)$ (generalized capital benefits).

Exercise 3.3

Let us assume in Exercise 2 a permanent disability model with transition rates given by

$$\mu_{*\diamond}(t) = 0.0279, \quad \mu_{*\dagger}(t) = 0.0229, \quad \mu_{\diamond\dagger}(t) = \mu_{*\dagger}(t).$$

Further, let the (constant) force of interest r_t be 3%. Calculate the total reserve $V_j(t, A)$ for all states j when the insured is 60 years old, i.e. $t = 30$. Calculate the total reserve $V_j(t, A)$ also when reactivation is allowed assuming $\mu_{\diamond *}(t) = 0.1\mu_{*\diamond}(t)$.