STK4500: Life Insurance and Finance

Exercise list 4

Exercise 4.1 (Term insurance in discrete time)

A 10 years old policy is issued to a life aged 50. The sum insured, payable at the end of the year of death, is 200 000\$ and premiums π are paid yearly throughout the term.

Assume that the intensity rate $r = 2.5\%$ and that

$$\mu_{*\dagger}(t) = 0.002 + 0.0005(t - 50).$$

Use the equivalence principle to calculate the annual premium π and the prospective reserves $V_*^+(t)$, $t = 0, \ldots, 10$.

Exercise 4.2 (Permanent disability insurance in discrete time)

Consider a permanent disability insurance. Let $x = 45$ years be the initial age of the insured and let 65 be the age at maturity. Further assume that $r = 3\%$ (intensity rate) and that the yearly disability pension is given by 12 000\$.

Compute the reserves of the disability pension for each of the following cases (see List 1, Exercise 5):

(i)

$$\mu_{*\diamond}(t) = 0.0279 \quad \mu_{*\dagger}(t) = 0.0229 \quad \mu_{\diamond\dagger}(t) = \mu_{*\dagger}(t).$$

(ii) Gompertz-Makeham model as follows

$$\mu_{*\diamond}(t) = a_1 + b_1 \exp(c_1 t) \quad \mu_{*\dagger}(t) = a_2 + b_2 \exp(c_2 t) \quad \mu_{\diamond\dagger}(t) = \mu_{*\dagger}(t),$$

where $a_1 = 4 \cdot 10^{-4}$, $b_1 = 3.4674 \cdot 10^{-6}$, $c_1 = 0.138155$, $a_2 = 5 \cdot 10^{-4}$, $b_2 = 7.5858 \cdot 10^{-5}$ and $c_2 = 0.087498$.

Exercise 4.3 (Endowment policy in discrete time)

Consider an endowment for a life aged 35 years with 60 years as age maturity. Suppose that $r = 3.5\%$ and that the death benefit is given by 250 000\$. The endowment is assumed to be 125 000\$. Further let

$$\mu_{*\dagger}(t) = 0.0015 + 0.0004(t - 35).$$

Calculate the annual premiums π based on the equivalence principle and the prospective reserves.

Exercise 4.4 (Discrete time vs. Continuous time)

Consider an endowment for a life aged 30 years with 65 years as age maturity. Suppose that $r = 3.5\%$ and that the death benefit is given by 200 000\$. The endowment is assumed to be 100 000\$. Consider the mortality rate $\mu_{*\dagger}(t)$ from Exercise 4.2 (ii).

Calculate the annual premiums π based on the equivalence principle and the prospective reserves both using discrete time formulas and continuous time formulas. Compare the results.

Exercise 4.5 (Price of bonds under the Vasicek model)

The price of a bond at time t with maturity $T > t$ is given by

$$P(t,T) = E\left[e^{-\int_t^T r_s ds} \middle| \mathcal{F}_t\right], \tag{0.1}$$

where $\mathcal{F} = \{\mathcal{F}_t\}_{t\in[0,T]}$ is the filtration generated by the process $r = \{r_t, t \in [0,T]\}$. In this exercise we assume that r is the solution of the following SDE

$$dr_t = a(b - r_t)dt + \sigma dW_t, \quad r_0 \in \mathbb{R}, \quad t \in [0,T]. \tag{0.2}$$

The aim of this exercise is to derive an explicit formula for the price of the bond in (0.1).

(i) Apply Itô's formula to $e^{at}r_t$ in order to show that

$$r_t = r_0 e^{-at} + b(1 - e^{-at}) + \sigma e^{-at}\int_0^t e^{as} dW_s, \quad t \geqslant 0, \tag{0.3}$$

and show that $r_t \sim N\left(r_0 e^{-at} + b(1 - e^{-at}), \frac{\sigma^2}{2a}(1 - e^{-2at})\right)$.

(ii) Use (0.2) and (0.3) starting at $r_s > 0$ to argue that $\int_s^t r_u du$ is also normally distributed and prove that

$$\mu_{s,t} := E\left[\int_s^t r_u du\right] = \frac{r_s - b}{a}(1 - e^{-a(t-s)}) + b(t - s),$$

and

$$\Sigma_{s,t}^2 := Var\left[\int_s^t r_u du\right] = \frac{\sigma^2}{2a^3}\left(3 - 2a(t-s) + e^{-2a(t-s)} - 4e^{-a(t-s)}\right).$$

(iii) Conclude that

$$E\left[e^{-\int_t^T r_s ds} \middle| \mathcal{F}_t\right] = e^{-\mu_{t,T} + \frac{1}{2}\Sigma_{t,T}^2}.$$