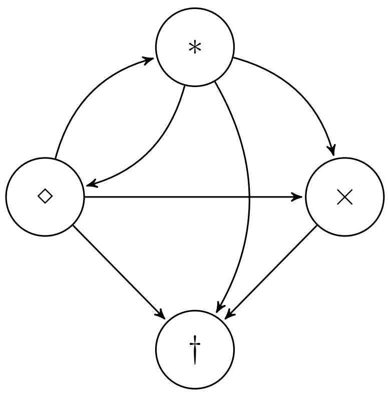
STK4500: Life Insurance and Finance

Exercise list 5

Exercise 5.1

Let us consider an insurance policy model which combines benefits of a disability income insurance (see Exercise list 2, Exercise 2) and benefits in the case of critical illness. The state space for the driving Markov chain X is $S = \{*, \diamond, \times, \dagger\}$ where $*$ means healthy, $\diamond$ means sick, $\times$ means critically ill (with no possibility of recovery) and $\dagger$ is death, i.e.



The transition intensities are given by

$$\begin{aligned}
&\mu_{*\diamond}(t) = a_1 + b_1 \exp(c_1 t), \quad \mu_{*\times}(t) = 0.05\mu_{*\diamond}(t), \quad \mu_{*\dagger}(t) = a_2 + b_2 \exp(c_2 t),\\
&\mu_{\diamond *}(t) = 0.1\mu_{*\diamond}(t), \quad \mu_{\diamond\times}(t) = \mu_{*\times}(t), \quad \mu_{\diamond\dagger}(t) = \mu_{*\dagger}(t),\\
&\mu_{\times\dagger}(t) = 1.2\mu_{*\dagger}(t).
\end{aligned}$$

where $a_1 = 4 \cdot 10^{-4}$, $b_1 = 3.4674 \cdot 10^{-6}$, $c_1 = 0.138155$, $a_2 = 5 \cdot 10^{-4}$, $b_2 = 7.5858 \cdot 10^{-5}$ and $c_2 = 0.087498$. Use the Euler approximation scheme with step size $h = \frac{1}{12}$ (1 month) to compute $p_{**}(x, x + 35)$ for an insured aged $x = 30$ years. You may also plot $t \mapsto p_{**}(30, t)$, $t \geqslant 30$.

Exercise 5.2

Consider a 10-years disability income insurance to a healthy life aged 60 years. Payments of 20 000\$ are provided by the insurer (continuously in time), while the insured is in the disabled state. A death benefit of 50 000\$ is immediately payable on death. Further, it is also required that premiums are payable continuously while the insured is in the healthy state. Assume that $r = 5\%$ (intensity rate) and that other expenses are ignored. The transition rates are given by

$$\mu_{*\diamond}(t) = a_1 + b_1 \exp(c_1 t) \quad \mu_{\diamond *}(t) = 0.1\mu_{*\diamond}(t), \quad \mu_{*\dagger}(t) = a_2 + b_2 \exp(c_2 t) \quad \mu_{\diamond\dagger}(t) = \mu_{*\dagger}(t),$$

where $a_1 = 4 \cdot 10^{-4}$, $b_1 = 3.4674 \cdot 10^{-6}$, $c_1 = 0.138155$, $a_2 = 5 \cdot 10^{-4}$, $b_2 = 7.5858 \cdot 10^{-5}$ and $c_2 = 0.087498$.

Calculate the (constant) annual premiums π for this policy.

Exercise 5.3

Again let us have a look at a disability income insurance with term two years issued to a healthy life aged 50. Premiums are paid continuously throughout the contract period while the policyholder is in the healthy state. Disability benefits are provided at the rate of 60 000\$ per year.

Assume that the following transition probabilities are applied to the insured:

$$p_{**}(50, 50+t) = \frac{2}{3}e^{-0.015t} + \frac{1}{3}e^{-0.01t},$$
$$p_{*\dagger}(50, 50+t) = 1 - e^{-0.01t}.$$

Use an intensity rate of 5% to compute the annual premiums π.