# STK4500: Life Insurance and Finance

Exercise list 8

## Exercise 8.1

Consider the process

$$M(t) := E[V|\mathcal{F}_t], \quad t \geqslant 0,$$

where $\mathcal{F} = \{\mathcal{F}_t\}_{t\geqslant 0}$ is a filtration (as a model for information flow over time) and $V$ is a random variable (e.g. present value of the insurer's liabilities) with $E[|V|] < \infty$. Verify that $M$ is a martingale with respect to $\mathcal{F}$.

## Exercise 8.2 (Generalized Black-Scholes model)

Let $Z = \{Z_t, t \in [0,T]\}$ be "market noise" modelled by a semimartingale with continuous paths. Assume that the price $S_t$ of a stock at time $t \in [0,T]$ is described by the following stochastic differential equation

$$S_t = S_0 + \int_0^t S_u dZ_u, \quad t \in [0,T].$$

(i) Find an explicit formula for the stock price process $S_t$, $t \in [0,T]$ by using Itô's formula.

(ii) Use the formula in $(i)$ to obtain a representation for $S_t$ in the case $Z_t = \int_0^t \mu ds + \int_0^t \sigma dB_s$ (classical Black-Scholes model) where $B$ is a Brownian motion, $\mu \in \mathbb{R}$ the mean return and $\sigma > 0$ the volatility.

## Exercise 8.3

Let $N = \{N_t, t \in [0,T]\}$ be a Poisson process with intensity $\lambda > 0$. Compute $\int_0^t N_{s-} dN_s$.