UiO : Department of Mathematics
University of Oslo

STK4500: Life insurance and finance

Conditional expectation (quick guide)

David R. Banos **Spring 2024**

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Conditional expectation: intuition

Consider (X, Y) a random vector (X and Y can be dependent).
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- Q: What is our best guess for X? A: Point estimates, e.g. $\mathbb{E}[X]$ or median, etc.

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- We denote by $\mathbb{E}[X|Y]$ the **conditional expectation** of X given Y.
- The concept of conditional expectation is based on the concept of *conditional probability*.
- Important: the conditional expectation **is** a random variable!
- Even more: $\mathbb{E}[X|Y]$ is a function of Y, i.e. there exists a function $f : \mathbb{R} \to \mathbb{R}$ such that

$$\mathbb{E}[X|Y] = f(Y).$$

Conditional expectation: geometric intuition

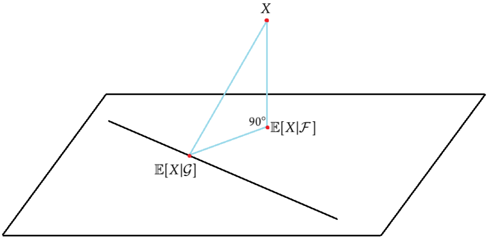

Figure: If $\mathcal{G} \subset \mathcal{F}$ then $\mathbb{E}[X|\mathcal{F}]$ is a better approximation of X than $\mathbb{E}[X|\mathcal{G}]$ (explain).

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as it should be.
Example: X result from die toss $A = \{\text{odd outcome}\}$. Then $\mathbb{E}[X] = 3.5$ but

$$\mathbb{E}[X|A] = \frac{1}{\mathbb{P}[A]}\mathbb{E}[1_A X] = \frac{1}{1/2}\sum_{x=1}^{6} x 1_{\{1,3,5\}}\frac{1}{6} = 2\,(1+3+5)\,\frac{1}{6} = 3.$$

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Conditional expectation: discrete/discrete

(X, Y) discrete with joint probability mass function $\mathbb{P}[X = x, Y = y]$. Then

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Example: $\mathbb{P}[X = x, Y = y] = \frac{x+y}{4}$ for $x, y = 0, 1$. Then $\mathbb{P}[Y = y] = \sum_x \mathbb{P}[X = x, Y = y] = \frac{2y+1}{4}$ and hence

$$\mathbb{P}[X = x|Y = y] = \frac{\mathbb{P}[X = x, Y = y]}{\mathbb{P}[Y = y]} = \frac{4}{2y+1}\frac{x+y}{4} = \frac{x+y}{2y+1}.$$

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Note that indeed $\mathbb{E}[X|Y]$ is a random variable:

$$\mathbb{E}[X|Y] = \frac{Y+1}{2Y+1}.$$

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Conditional expectation: continuous/discrete

(X, Y) with joint distribution function $\mathbb{P}[X \leq x, Y = y]$. Then

$$\mathbb{E}[X|Y = y] = \int_x x f_{X|Y}(x|y) dx,$$

where $f_{X|Y}(x|y)$ is the conditional density of X given y, that is the derivative of the conditional distribution function $F_{X|Y}(x|y) := \mathbb{P}[X \leq x|Y = y]$.

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Example: Let (X, Y) with joint distribution

$$\mathbb{P}[X \leq x, Y = y] = \frac{1}{2} \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-y)^2}{2}} dz, \quad (x, y) \in \mathbb{R} \times \{0, 1\}.$$

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Observation: $X|Y \sim N(Y, 1)$ and Y is Bernoulli with parameter $1/2$. So X has distribution (explain on blackboard)

$$\mathbb{P}[X \leq x|Y = y] = \frac{\mathbb{P}[X \leq x, Y = y]}{\mathbb{P}[Y = y]} = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-y)^2}{2}} dz.$$

Hence,

$$\mathbb{E}[X|Y = y] = \int_{\mathbb{R}} x f_{X|Y}(x|y) dx = \int_{\mathbb{R}} x \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-y)^2}{2}} dx = y.$$

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Conditional expectation: discrete/continuous

X discrete and Y continuous. Let (X, Y) with joint distribution function $\mathbb{P}[X = x, Y \leq y]$. Then

$$\mathbb{E}[X|Y = y] = \sum_x x\mathbb{P}[X = x|Y = y],$$

where

$$\mathbb{P}[X = x|Y = y] = \frac{f_{Y|X}(y|x)\mathbb{P}[X = x]}{f_Y(y)}$$

and where $f_{Y|X}(y|x)$ is the conditional density of Y given X and f_Y is the density of Y.

Example: Let (X, Y) with joint distribution

$$\mathbb{P}[X = x, Y = y] = \left(y1_{\{x=1\}} + (1 - y)1_{\{x=0\}}\right) f_Y(y),$$

where f_Y is the density function of Y.
Then find $\mathbb{E}[X|Y]$. You should obtain: $\mathbb{E}[X|Y] = Y$.

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Definition of conditional expectation

(X, Y) with joint density function $f_{X,Y}(x,y) = \frac{d^2}{dxdy}\mathbb{P}[X \leq x, Y \leq y]$. Then

$$\mathbb{E}[X|Y=y] = \int_x x f_{X|Y}(x|y)dx,$$

where $f_{X|Y}(x|y)$ is the conditional density of X given $Y = y$ defined as

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}.$$

Example: Let (X, Y) with joint distribution $f_{X,Y}(x,y) = \frac{\lambda}{y}e^{-\lambda y}1_{[0,y]}(x)$, $(x,y) \in [0,\infty)^2$. Show that $\mathbb{E}[X|Y] = \frac{Y}{2}$ and if you want: $\mathbb{E}[Y|X] = \frac{e^{-\lambda X}}{\int_X^\infty \frac{\lambda}{y}e^{-\lambda y}dy}$.

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Properties of conditional expectation

X, Y and Z are random variables.

- It is linear: $\mathbb{E}[aX + bY|Z] = a\mathbb{E}[X|Z] + b\mathbb{E}[Y|Z]$.

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- If X is a function of Y, i.e. $X = f(Y)$ then $\mathbb{E}[X|Y] = X$.
- If knowing Y implies knowing Z then $\mathbb{E}[\mathbb{E}[X|Y]|Z] = \mathbb{E}[X|Z]$.

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Conditional expectation for our needs

In insurance we have $V(t)$, $V^{-}(t)$ and $V^{+}(t)$ random variables and $Z(t)$ the state of the insured.

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Remember that $Z(t)$ is a discrete random variable for every fixed t.

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The random variables $V(t)$, $V^{-}(t)$ and $V^{+}(t)$ depend on $Z(t)$, $0 \leq t \leq T$.
Remember that $Z(t)$ is a discrete random variable for every fixed t.
At a given time t, one can condition on the whole family of random variables $\{Z(s), 0 \leq s \leq t\}$ until time t. One would typically write

$$\mathbb{E}[V(t)|Z(s), 0 \leq s \leq t]$$

meaning: *the conditional expectation of the present value* $V(t)$, *given that we fully know all the states of the insured from* 0 *to a hypothetical time* t.

Conditional expectation for our needs

Since $V(t) = V^{-}(t) + V^{+}(t)$ then

$$\mathbb{E}[V(t)|Z(s), 0 \leq s \leq t] = \mathbb{E}[V^{-}(t)|Z(s), 0 \leq s \leq t] + \mathbb{E}[V^{+}(t)|Z(s), 0 \leq s \leq t].$$

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For $V^{-}(t)$ we have

$$V^{-}(t) = \frac{1}{v(t)} \left(\sum_{i} \int_{[0,t]} I_i^Z(s) da_i(s) + \sum_{i,j:j \neq i} \int_{[0,t]} a_{ij}(s) dN_{ij}^Z(s) \right).$$

Conditional expectation for our needs

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Note that $V^-(t)$ depends only on $Z(s)$ for $0 \le s \le t$ through $I_i^Z(s)$ and $dN_{ij}^Z(s)$ on $[0, t]$. Hence, knowing $Z(s)$ for all $0 \le s \le t$ would imply knowing $V^-(t)$ fully! Thus,

$$\mathbb{E}[V^-(t)|Z(s), 0 \le s \le t] = V^-(t).$$

Conditional expectation for our needs

On the other hand, for $V^+(t)$ we have

$$V^+(t) = \frac{1}{v(t)} \left(\sum_i \int_{(t,\infty)} I_i^Z(s)da_i(s) + \sum_{i,j:j\neq i} \int_{(t,\infty)} a_{ij}(s)dN_{ij}^Z(s) \right).$$

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Note that $V^+(t)$ depends only on the future states $Z(s)$ for $s \in (t,\infty)$ through $I_i^Z(s)$ and $dN_{ij}^Z(s)$ on (t,∞). However, we only know the past, i.e. $Z(s)$ for all $0 \leq s \leq t$ and not $Z(s)$, $s \in (t,\infty)$. Hence,

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$$\mathbb{E}[V^+(t)|Z(s), 0 \leq s \leq t] \neq V^+(t).$$

$V^+(t)$ is a functional of the future states $Z(s)$, $s \geq t$ and we know $Z(s)$ for all past times $s \in [0,t]$. Since Z is Markov we can conclude with the following important property:

$$\mathbb{E}[V^+(t)|Z(s), 0 \leq s \leq t] = \mathbb{E}[V^+(t)|Z(t)].$$

This is known as the Markov property! We only need to use the last state $Z(t)$ to guess the future $V^+(t)$.

Conditional expectation for our needs

In a summary

$$\mathbb{E}[V(t)|Z(s), 0 \leq s \leq t] = V^{-}(t) + \mathbb{E}[V^{+}(t)|Z(t)].$$

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In this course, we will focus entirely in computing $\mathbb{E}[V^{+}(t)|Z(t)]$ which is a conditional expectation of a random variable $V^{+}(t)$ given a (discrete) random variable $Z(t)$. See previous slides.

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Recall that $Z(t)$ takes values in $\mathcal{Z}$ hence we can compute

$$V_i^+(t) \triangleq \mathbb{E}[V^+(t)|Z(t) = i].$$

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Then

$$\mathbb{E}[V^{+}(t)|Z(t)] = V_{Z(t)}^{+}(t)$$

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Then

$$\mathbb{E}[V^+(t)|Z(t)] = V_{Z(t)}^+(t)$$

and the following relation follows

$$V_{Z(t)}^+(t) = \sum_{i \in \mathcal{Z}} V_i^+(t) I_i^Z(t), \quad \text{(explain)}.$$

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