UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in: STK4500/9500 — Life Insurance and Finance

Day of examination: 10th June 2022

Examination hours: 3:00 pm – 7:00 pm

This problem set consists of 2 pages.

Appendices: None

Permitted aids: Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

This exam consists of four problems. The first one is of theoretical nature, while the three remaining problems are more applied. Make sure to be **precise** and **rigorous** when stating mathematical results and formulae. Please, **write clearly** and avoid crossing out.

Problem 1 Theory (weight 4 points)

State and prove Thiele's ordinary differential equation for the prospective reserve (when everything is deterministic except for the state of the insured). You may assume for simplicity that the policy functions a_i are a.e. differentiable and continuous. Comment briefly on how you would solve the equation numerically.

Problem 2 Endowment policy (weight 4 points)

Let $X = \{X_t\}_{t\geq 0}$ be a regular continuous time Markov process with state space $S = \{*, \dagger\}$ where $X_t = *$ means that the insured is active at time t and $X_t = \dagger$ means that the insured is deceased at time t. Denote by $\mu := \mu_{*\dagger} > 0$, $t \geq 0$ the intensity rate of transitioning from state $*$ to state $\dagger$ which, for simplicity, we take to be constant (NB! this is far from realistic, but this is a written exam without computers).

Consider an endowment insurance contract for a $x_0 \geq 0$ years old person with expiry at age $x_0 + T$, $T \geq 0$ being the contract length. This contract pays the benefit B in case that the insured survives by the end of the contract and a death benefit C in case the insured dies during the period of the contract. Assume constant interest rate r.

(a) Show that the value of this insurance at any time $t \in [0, T]$ is given

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by

$$V_*^+(t,A) = Be^{-(r+\mu)(T-t)} + C\frac{\mu}{r+\mu}\left(1 - e^{-(r+\mu)(T-t)}\right), \quad t \in [t_0, T].$$

(b) Compute the (fair) single premium at the beginning of the contract and the (fair) yearly premiums.

Problem 3 Surrender policy (weight 4 points)

A healthy life of age x_0 writes an endowment insurance with contract length $T > 0$. The death benefit corresponds to the current prospective reserve at the time of death and the endowment benefit is a fixed lump sum of E units. The policyholder pays a single π_0 at the beginning of the contract.

We allow the policyholder to surrender the policy at any time. In such case, we pay them a benefit corresponding to the current prospective reserve at the surrendering time with a penalty, i.e. $(1-\varepsilon)$, $\varepsilon \in [0,1]$, multiplied by the prospective reserve at the surrendering time.

Let $\mathcal{S} = \{*, \dagger, s\}$ be the states of this insurance being $*$ alive, $\dagger$ deceased and s surrendered. Let $\mu_{*\dagger}$ and μ_{*s} denote the transition rates to *deceased* and *surrendered*, respectively, hereby assumed to be constant.

Prove that the prospective reserve for this policy coincides with the prospective reserve of a pure endowment policy with interest rate r and with respect to a Markov chain X^ε with state space $\mathcal{S}^\varepsilon = \{*, \dagger^\varepsilon\}$ with $\mu_{*\dagger^\varepsilon} = \varepsilon\mu_{*s}$. Could you give an interpretation of why?

Problem 4 Unit-linked policy (weight 4 points)

Consider a (continuous) permanent disability model with (constant) $\mu_{*\diamond} = 0.0279$, $\mu_{*\dagger} = 0.0229$ and $\mu_{\diamond\dagger} = \mu_{*\dagger} = 0.0229$ for a life of age $x_0 = 30$. The term of the policy is $T = 10$ years. The disability pension is financed by the value of a fund described by a stochastic process $S = \{S_t, t \in [0,T]\}$.

Assuming that the initial investment is $S_0 = 100\,000$ and that interest rate is $r = 3\%$, compute the exact numerical value of the single premium π_0 of this insurance. Hint: solving Kolmogorov's equations with constant rates one can arrive at $p_{**}(t,s) = e^{-(\mu_{*\diamond}+\mu_{*\dagger})(s-t)}$, $p_{\diamond\diamond}(t,s) = e^{-\mu_{\diamond\dagger}(s-t)}$ and $p_{*\diamond}(t,s) = \frac{\mu_{*\diamond}}{\mu_{*\diamond}+\mu_{*\dagger}-\mu_{\diamond\dagger}}\left[e^{-\mu_{\diamond\dagger}(s-t)} - e^{-(\mu_{*\diamond}+\mu_{*\dagger})(s-t)}\right]$.

GOOD LUCK!

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