Home exam 2005 for STK4520

Introduction

There are two separate exercises. Both must be analysed and reported. The presentation should be brief, concentrate on the essentials and yet explain assumptions, conditions and methods so that other analysts can understand in detail how things have been carried out. It may well be a good idea to split the texts in a main part that can be read without actuarial expertise and more a technical part (an appendix?). But there are many ways to do it as there are also more than one way to solve the problems. You are expected to come up with methods that make sense. Language: Norwegian or English.

Students will be judged on the basis of their reports (that must be written individually!) and an oral examination (45 minutes) where their work will be discussed. The time table will be found on the web. The oral session will start by a brief presentation by the students themselves and then questions will be asked. Most of the discussion will focus on the report and the two problems posed, but it is also possible that it may wander into related issues a little (but not much) on the outside of what has actually been reported.

Return electronically (preferably pdf format) to **erikb@math.uio.no** within December 13, 16.00. Questions about the exercises can be sent to the same address.

Exercise 1

This problem is patterned on a current legal dispute that will be argued before the city court of Oslo next year. Data have been masked (they are not quite the real ones) and one detail has been simplified. You are to pretend that the problem goes into a real tax issue and write a report where conditions and conclusion can be understood by laymen so far as this is possible.

Oil companies are entitled to deduct insurance premia from their income tax return. Over a few years that accumulates into a lot of money. Such contracts are rarely placed in the ordinary market. Usually oil companies set up captives they own themselves to carry these risks. That is for taxation reasons and are perfectly legal, but it does create problems for the tax deduction. What is the annual premium exactly? You are to report on that for company C based on a historical record of 20 accidents among its oil rigs over the preceding 6 years. Their size (in million NOK) are

| 0.3 | 1.1 | 1.2 | 1.5 | 7.2 | 10.5 | 10.8 | 12.4 | 20.6 | 21.4 |
| 26.6 | 30.2 | 30.6 | 40.2 | 60.3 | 64.5 | 68.2 | 87.2 | 135.6 | 210.3 |

The number of rigs has not changed at all over this period.

Very often oil exploration are joint ventures where a given company only owns some share of the rig. In this case company C have 37% of all rigs and so only needs reimbursement for only 37% of the size of a given accident. The policy issued by the captive also contain clauses of deductibles and maximum responsibility. If the company C share of the damage is less than 3 million NOK, nothing is received from the insurer. Over that threshold 3 million is subtracted. There is also a ceiling on what the company can receive from the captive for a single accident. The maximum is 50 million.

Estimate the annual pure premium for the responsibility of the captive and also the actual premium when you assume a premium loading of 25%. Also report on the uncertainty of the assessment. Include technical assumptions too so that others can understand how you have attacked the problem

Exercise 2
One issue that attracts much current interest in the Norwegian life insurance industry is defined benefit pension schemes where people build up their own pension, carry themselves all financial risk, but benefit from the management expertise of companies. At retirement age the capital accumulated is used to buy a life annuity for the invidual at a fixed interest rate $r03\%$. The problem posed is the uncertainty of the annuity.

We shall examine the issue for an individual that enters the scheme at the age of 37 years (start of it) and retires at 67 (the first year the annuity is paid). In case of early death (before retirement) the capital is taken over by benficiaries like spouses or children. All calculations are annual. The contributions of the individual are fixed at 35000 NOK each year (again made in the beginning of the year). Survival probabilities are

$$\log(p_l) = -0.0009 - 0.0000462 e^{0.090767 \times l}$$

where p_l is the probability of living through the coming year given that the indvidual are l years at the start of it. Investments will be in the money market (earning annual return r_k) or in equities (now the return is R_k). Their models are

$$r_k = \xi_1 \exp\left(-\frac{\sigma_1^2}{2(1-a_1^2)} + X_{1k}\right) \qquad \text{where} \qquad X_{1k} = a_1 X_{1k-1} + \sigma_1 \varepsilon_{1k}$$

and

$$R_k = (1+\xi_2) \exp\left(-\frac{\sigma_2^2}{2} + X_{2k}\right) - 1 \qquad X_{2k} = \sigma_2 \varepsilon_{2k}.$$

Here $(\varepsilon_{1k}, \varepsilon_{2k})$ is an independent sequence of normal pairs with mean zero and variance 1. It is assumed that

$$\text{cor}(\varepsilon_{1k}, \varepsilon_{2k}) = -0.2 \qquad \text{(at the same time)}.$$

Other parameters used in this exercise are

$$\xi_1 = 4\%, \;\; \sigma_1 = 0.25 \;\; a_1 = 0.6 \qquad \text{and} \qquad \xi_2 = 8\%, \;\; \sigma_2 = 0.1.$$

The interest rate at the start (at $k = 0$) is $r_0 = 4\%$. Companies managing such an account will charge for their services. How much is in Norway too early to say. We shall assume both 0.5% and 1% annual of the current value of the account up to retirement.

Report on how much the annuity is going to vary under the following three investment strategies:

- All placed in the money market
- All placed in the equity market

- 30% in equity and rest in money products[1]

Also report on how sensitively the annuity depends on the management cost. Finally, try to reflect on whether the analysis should be modfied if the insurance company rather then family takes over the accumulated saving in case of an early death before retirement.

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[1] This means that each year is 30% of the annual contributions is placed in equity and the rest in the money market. No reblancing of the portfolio is carried out