Exercise 11

Equity-linked life insurance contracts allow the company to transfer the financial risk to the customers. In practice such contracts are often organized by the clients owning a share of some fund all policy holders having contributed to, but the arrangement can equally well be analysed by tracking the movements of an individual account. This means that the value V_k of the account at time k is built up according to the recursion

$$V_k = (1 + R_k)V_{k-1} + \pi_k, \qquad k = 1, \ldots, K.$$

Here R_k is the financial return in period k, and π_k a contribution into the account. Assume that

$$V_0 = \pi, \qquad \pi_k = \pi, \text{ for } k < K, \qquad \pi_K = 0.$$

It is assmed that the account is invested in the stock market. The standard model is that the return R_k is log-normally distributed, i.e.

$$R_k = \exp(\xi + \sigma\varepsilon_k)$$

where the error terms ε_k are standard Gaussian random variables. The parameters are consistant with an annual time increment; i.e.

$$\xi = 0.10, \qquad \sigma = 0.25.$$

The contribution is annual time increment; i.e.

$$\pi = 12000 \text{ NOK} \qquad \text{and } K = 20,$$

so that twelve thousand NOK is contributed at the start of each year, staring 20 years before retirement age.

a) Implement a simulation program that computes the probability distribution of V_K at the start of retirement age.

b) Copute relevant quantities that convey average and variation of the account at that time.

c) Also use the program when $\xi = 0.04$ and $\sigma = 0$ (riskfree investments) and compare with the results in b).

Suppose the account V_K is to be used for a life annuity at interest $r = 0.02$. The risky part of the investments is now removed and the company promises a fixed return for as long as the insured stays alive. The pay-out is at the start of each year. The mortalities are those in Exercise 5, scaled up so that the average length of life is 80 years.

d) Write down a formula for the annual advance payment when the expected present value at time K of the income stream to the policy holder is exactly V_K.

e) Modify the simulation program so that you can read off the variation in the annual pension.

f) Under what circumstances will the company earn money on the contract? They have received no fee during the build-up of the account.