Exercise 12

This is a follow-up of exercise 7, but we shall now take into consideration **inflation**. Adding to the model in exercise 7 is then a price level process of the form

$$P_k = P_{k-1}\exp(I_k) \qquad P_0 = 1$$

where

$$I_k = \xi_I + Z_{Ik}, \qquad Z_{Ik} = a_I Z_{Ik-1} + \sigma_I \varepsilon_{Ik}$$

This is the model proposed by David Wilkie on the basis of studying the twentieth century. Wilkie's estimates are

$$\xi = 4.7\% \qquad a_I = 0.58 \qquad \sigma_I = 4\%.$$

We ignore their statistical uncertainty (which is considerable). As usual the process $\{\varepsilon_{Ik}\}$ consists of independent variables with mean zero and standard deviation 1.

a) Simulate inflation and price level 50 years ahead. Repeat and judge the variability.

b) If V_K is the value of the account of exercise 7, explain why V_K/P_K is the real value of the account.

To get hold of this quantity we must simulate the development of both the account and the price level. We assume that the input into the account is the same quantity as in exercise 7, but in real terms.

c) Explain what that means. Write down the mathematical expression.

Another issue is that the return from the stock market and the inflation is correlated. If inflation is high, then so the return might be high too (this is one of the advantages of equity investments). One way to buld this in is to simulate $\{\varepsilon_k, \varepsilon_{Ik}\}$ as a sequence of correlated pairs, but still independent from one period of time to another. We might achieve this by taking

$$\varepsilon_{Ik} = \rho\varepsilon_k + \sqrt{1-\rho^2}\eta_k$$

where $\{\eta_k\}$ is another independent, Gaussian process with mean zero and variance one. All η_l are to be independent from all ε_k for all $l.$ and k

d) Explain why this trick achievs that the models for equity returns and inflation still are what they were, but that now inflation and returns depend on each other.

e) Simulate the real value V_K/P_K of the account when $K = 20$. Describe the variation. Use $\rho = 0.5$ and $\rho = 0.8$.

f) Repeat the exercise for the scheme of guaranteed fixed interest rate in exercise 7. You are now to take into consideration that inflation reduces the value of your account. Compare with the bolder equity investment program in e).