

### Exercise 2.1

Prove the floor formula in Chapter 14 (in Section 1.6).

### Exercise 2.2

Suppose interest rate follows the Black-Karlsruhe model with  $\xi = 4\%$ ,  $\sigma = 0.25$  and  $a = 0.7$ . The initial rate is  $r = 8\%$ .

a) Use the program in Exercise 1.3 to compute the theoretical bond price and the theoretical interest rate curve.

b) What's their mathematical relationship?

If you haven't done it yourself, an approximate answer to a) is (in %)

Time	1	2	3	4	5	6	7	8	9	10
Discount (%)	94.347	89.574	85.389	81.642	78.208	75.028	72.049	69.244	66.574	64.016
Yield curve (%)	5.992	5.660	5.406	5.202	5.039	4.905	4.795	4.701	4.624	4.561

c) Compute the steady-state volatility for  $r_k$  when the model is Black-Karlsruhe model with the parameters above [Answer: 0.01015 ]

Suppose  $\sigma_k = 0.0105\sqrt{1 - 0.7^{2k}}$  is the volatility function for the interest rate under the risk neutral model.

d) Explain how the floorlet prices are determined from the information given.

e) Write a program that computes the floor price when the derivative is annual and lasts from 1 to 10 years. What price is charged for for the entire floor?

f) Suppose your initial capital is  $v_0$  and that you purchase a floor, as above, to protect financial earnings. You use part of the capital to pay for the derivative. How much is left after expenses for the floor has been deducted?

### Exercise 2.3

Write a program that computes the swap rate from the information in Exercise 2.2. What is the numerical value for it?