Exercise 6.1

We are in this exercise going to examine how hetereogenity in claim frequency influences the reserve when the claim size is the same for the whole portfolio. Let

$$N^{\text{po}} = \sum_{j=1}^{J} N_j$$

be the total number of claims against the portfolio. Here $N_1, \ldots, N_J$ are the claims against the individual policies. It is assumed that each N_j is Poisson distributed with parameters $T\mu_1, T\ldots, T\mu_J$ where T is the period of time in question. The intensites $\mu_1, \ldots, \mu_J$ may be both fixed or stochastic. The total claim against the portfolio is then

$$X^{\text{po}} = \sum_{i=1}^{N^{\text{po}}} X_i$$

where X_i is the size of the i'th claim. It is assumed that all claims are log-normal distributed.

a) Implement a computer program that samples the portfolio claim when the intensitities $\mu_j = \xi$, a fixed parameter that applies to all policies.

b) Repeat a) when all μ_j are drawn independently of each other from a log-normal distribution with mean ξ and standard deviation τ.

c) Repeat a) when again all $\mu_1 = \ldots = \mu_J = \mu$ are equal, but when no the *common value* μ is generated from a log-normal with mean ξ and standard deviation τ.

Numerical experiement are run when

- $J = 1000$ and $J = 100000$
- $T = 1$, $\xi = 0.05$ and $\tau = 0.03$
- The log-normal for X_i has mean 1 and standard deviation 1

d) Compute the upper 1% and 5% percentiles for the total portfolio claim under the three models described in a), b), and c) and the parameters listed above.

d) Draw your conclusions: What have you learnt through these experiements.

You may need the following: If $\log(Z)$ is normal (η, σ),then

$$E(Z) = \exp(\eta + \sigma^2/2), \qquad \text{var}(Z) = \exp(2\eta + \sigma^2)\{\exp(\sigma^2) - 1\}$$

You adapt the parameters η and σ corresponding to given mean and standard deviations of Z trough these formulas. If you need a sampling procedure for normal variables you may utilize Box-Muller's algorithm. If U_1 and U_2 are indpendent and uniform, then

$$X_1 = \sqrt{-2\log(U_1)}\sin(2\pi U_2)$$

$$X_2 = \sqrt{-2\log(U_1)}\cos(2\pi U_2)$$

are independent and normal $(0, 1)$.