Exercise 9

The purpose of this exercise is two-fold, firstly to implement a pension scheme and secondly to evaluate the effect of people living longer.

Let N_x be number of members of x years of age. Their average annual benefit when they eventually draw a pension is S_x. The entire pension is paid out at the start of the year. We need the annual mortality p_x; i.e. p_x is the probability of dying within the year at age x. The probability of surviving the coming k years is then

$$_kq_x = (1 - p_x)(1 - p_{x+1}) \cdots (1 - p_{x+k-1}).$$

With retirement age a_0 the expected present value of the payments is

$$E(\text{PV}) = \sum_x \sum_{k \geq \max(a_0 - x, 0)} \frac{1}{(1+r)^k} (N_x \, _kq_x) S_x,$$

where r is interest.

a) Justify this formula

As baseline mortalities we shall assume that

$$p_x^0 = a + bc^x$$

where

$$a = 0.0009, \qquad b = 0.000044, \qquad c = 1.10154$$

It is first assumed that $p_x = p_x^0$.

b) Make a program that computes the

$$_kp_x = _{k-1}q_x p_{x+k-1}$$

which is the probability that a person of x years dies k years later.

c) Create a portfolio of one million people that all entered the pension scheme at the age of 40; i.e. lay out the numbers N_x of the pension scheme. [Hint: You multiply the probabilities $_kq_{40}$ with one million up to some maximum age.]

Suppose

$$S_x = 0.2 \text{ million NOK}, \qquad a_0 = 67, \qquad r = 4\%.$$

d) Compute the expected present value of the payments of the pension scheme constructed in c).

The second half of the exercise follows the first, except for the mortalities now being

$$p_x = p_x^0 \exp(-\theta),$$

where θ is some parameter. Positive θ means that people live longer. Introduce

$$L_x = \sum_{k \geq 0} k \, {}_k p_x.$$

e) Justify that this is the expected remaining length of a person that has become x years of age.

f) Recompute the expected present value of the payments when θ is varied and also compute L_{20}. Make a plot of the two quantites against each other.

g) Use the plot of the preceding point to find out how much the expected payment goes up when L_{20} increases by five years.